Midterm Examination
Administered: Friday, October 10, 2003

Problem (1)
Consider the first order linear ordinary differential equation. Assume the argument of the cosine is in radians.

$$\frac{dy}{dt} = f(y,t) = \cos(t)y + 2t^2 \tag{1}$$

with the initial condition

$$y(t_o = 1) = 3 \tag{2}$$

The second order numerical method to solve this problem is given by

$$y_i = y_{i-1} + (t_i - t_{i-1})\frac{1}{2}\left[f\left(y_{i-1}, t_{i-1}\right) + f\left(y_i, t_i\right)\right] \tag{3}$$

(a) Use Heun's method to approximate y at $t = 1.1$. (Use one interval of size Δt=0.1)
(b) Solve part (a) again but take advantage of the linearity of the ODE to avoid the approximation inherent in Heun's method.
(c) Explain why the answers in (a) and (b) are different? Which answer is more accurate?

Problem (2)
Consider the system of linear algebraic equations:

$$4x_1 - 5x_2 = 2$$
$$5x_1 - 6x_2 = -1$$

Demonstrate that the multivariate Newton-Raphson will exactly solve a system of linear algebraic equations in one iteration. Use the initial guess of your choice.

Problem (3)
Consider the system of two linear ODES.

$$\frac{dx_1}{dt} = x_1 + 9x_2$$
$$\frac{dx_2}{dt} = 5x_1 - 6x_2$$

Determine the type of critical point and the stability.