Homework Assignment Number Six Solutions

We intend to purchase a liquid as a raw material for a material we are designing. Two vendors offer us samples of their product and a statistic sheet. We run the samples in our own labs and come up with the following data:

Vendor 1

sample #	outcome
1	2.30
2	2.49
3	2.05
4	2.40
5	2.18
6	2.12
7	2.38
8	2.39
9	2.40
10	2.46
11	2.19
12	2.04
13	2.43
14	2.34
15	2.19
16	2.12

Vendor 2

sample #	outcome
1	2.49
2	1.98
3	2.18
4	2.36
5	2.47
6	2.36
7	1.82
8	1.88
9	1.87
10	1.87

Vendor Specification Claims:

Vendor 1: $\mu = 2.0$ and $\sigma^2 = 0.05$, $\sigma = 0.2236$

Vendor 2: $\mu = 2.3$ and $\sigma^2 = 0.12$, $\sigma = 0.3464$

Sample statistics

$$n_1 = 16 \quad \bar{x}_1 = \frac{1}{16}\sum_{i=1}^{16} x_i = 2.280 \qquad s_1^{\,2} = \frac{1}{16}\sum_{i=1}^{16}\left[(x_i - \bar{x}_1)^2\right] = 0.0229 \qquad s_1 = 0.1513$$

$$n_2 = 10 \qquad \bar{x}_2 = \frac{1}{10}\sum_{i=1}^{10} x_i = 2.128 \qquad s_2^{\,2} = \frac{1}{10}\sum_{i=1}^{10}\left[(x_i - \bar{x}_2)^2\right] = 0.0744 \qquad s_2 = 0.2728$$

Problem 1.

Determine a 95% confidence interval on the mean of sample 1. Use the value of the population variance given. Is the given population mean legitimate?

$$P(\bar{X} + z_\alpha \frac{\sigma}{\sqrt{n}} < \mu < \bar{X} - z_\alpha \frac{\sigma}{\sqrt{n}}) = 1 - 2\alpha$$

$$P(2.280 - 1.96\frac{0.2236}{\sqrt{16}} < \mu < 2.280 + 1.96\frac{0.2236}{\sqrt{16}}) = 0.95$$

$$P(2.1704 < \mu < 2.3896) = 0.95$$

The claimed population mean of 2.0 does not fall within a 95% confidence interval.

The z values could be obtained either from a table or from MatLab. The MatLab commands are given below.

```
>> z = icdf('normal',0.025,0,1)

z =  -1.959963984540055
```

and

```
>> z = icdf('normal',0.975,0,1)

z =  1.959963984540054
```

Problem 2.

Determine a 95% confidence interval on the difference of means between samples 1 and 2. Use the values of the population variance given. Is the difference between the given population means legitimate?

$$P\left[\left(\bar{X}_1-\bar{X}_2\right)+z_\alpha\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}<\left(\mu_1-\mu_2\right)<\left(\bar{X}_1-\bar{X}_2\right)-z_\alpha\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}\right]=1-2\alpha$$

$$P\left[(2.280-2.128)-1.96\sqrt{\frac{0.05}{16}+\frac{.12}{10}}<\left(\mu_1-\mu_2\right)<(2.280-2.128)+1.96\sqrt{\frac{0.05}{16}+\frac{.12}{10}}\right]=0.95$$

$$P[-0.0890<\left(\mu_1-\mu_2\right)<0.3930]=0.95$$

The claimed difference of population means of -0.3 does not fall within a 95% confidence interval.

The z values used in this problem could be obtained either from a table or from MatLab. The MatLab commands are given below.

```
>> z = icdf('normal',0.025,0,1)

z =  -1.959963984540055
```

and

```
>> z = icdf('normal',0.975,0,1)

z =  1.959963984540054
```

Problem 3.

Determine a 95% confidence interval on the mean of sample 1. Assume the given values of the population variances are suspect and not to be trusted. Is the given population mean legitimate?

$$v=n-1=15$$

$$P(\bar{X}-t_\alpha\frac{s}{\sqrt{n}}<\mu<\bar{X}+t_\alpha\frac{s}{\sqrt{n}})=1-2\alpha$$

$$P(2.280-2.131\frac{0.1513}{\sqrt{16}}<\mu<2.280+2.131\frac{0.1513}{\sqrt{16}})=0.95$$

$$P(2.1994<\mu<2.3606)=0.95$$

The claimed population mean of 2.0 does not fall within a 95% confidence interval.

The t values used in this problem could be obtained either from a table or from MatLab. The MatLab commands are given below.

```
>> t = icdf('t',0.025,15)

t =  -2.131449545559775
```

and for the upper limit

```
>> t = icdf('t',0.975,15)

t =  2.131449545559775
```

Problem 4.

Determine a 95% confidence interval on the difference of means between samples 1 and 2. Assume the given values of the population variances are suspect and not to be trusted. Is the difference between the given population means legitimate?

$$v = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\left[\left(\frac{s_1^2}{n_1}\right)^2 \Big/ (n_1 - 1)\right] + \left[\left(\frac{s_2^2}{n_2}\right)^2 \Big/ (n_2 - 1)\right]} \text{ if } \sigma_1 \neq \sigma_2$$

$$v = \frac{\left(\frac{0.0229}{16} + \frac{0.0744}{10}\right)^2}{\left[\left(\frac{0.0229}{16}\right)^2 \Big/ (16 - 1)\right] + \left[\left(\frac{0.0744}{10}\right)^2 \Big/ (10 - 1)\right]} = 12.5178 \approx 13$$

confidence interval:

$$P\left[\begin{array}{l}(2.280 - 2.128) - 2.160\sqrt{\frac{0.0229}{16} + \frac{0.0744}{10}} < \\ \qquad (\mu_1 - \mu_2) < (2.280 - 2.128) - 2.160\sqrt{\frac{0.0229}{16} + \frac{0.0744}{10}}\end{array}\right] = 0.95$$

$$P[-0.0514 < (\mu_1 - \mu_2) < 0.3554] = 0.95$$

The claimed difference of population means of -0.30 does not fall within a 95% confidence interval.

The t values used in this problem could be obtained either from a table or from MatLab. The MatLab commands are given below.

```
>> t = icdf('t',0.025,13)

t =  -2.160368656462786
```

and for the upper limit

```
>> t = icdf('t',0.975,13)

t =  2.160368656462786
```

Problem 5.

Determine a 95% confidence interval on the variance of sample 1. Is the given population variance legitimate?

$$v = n - 1 = 15$$

$$P\left[\frac{(n-1)s^2}{\chi^2_{1-\alpha}} < \sigma^2 < \frac{(n-1)s^2}{\chi^2_{\alpha}}\right] = 1 - 2\alpha$$

$$P\left[\frac{(16-1)0.0229}{27.488} < \sigma^2 < \frac{(16-1)0.0229}{6.262}\right] = 0.95$$

$$P\left[0.0125 < \sigma^2 < 0.0547\right] = 0.95$$

The claimed population variance $\sigma^2 = 0.05$ does fall within this interval.

The χ^2 values used in this problem could be obtained either from a table or from MatLab. The MatLab commands are given below.

```
>> chi2 = icdf('chi2',0.025,15)

chi2 =   6.262137795043251
```

and for the upper limit

```
>> chi2 = icdf('chi2',0.975,15)

chi2 =  27.488392863442972
```

Problem 6.

Determine a 98% confidence interval on the ratio of variance of samples 1 & 2. Is the ratio of the given population variances legitimate?

$$v_1 = n_1 - 1 = 15,\ v_2 = n_2 - 1 = 9$$

$$P\left[\frac{S_1^2}{S_2^2}\frac{1}{f_{1-\alpha}(v_{1,}v_2)} < \frac{\sigma_1^2}{\sigma_2^2} < \frac{S_1^2}{S_2^2}\frac{1}{f_{\alpha}(v_{1,}v_2)}\right] = 1 - 2\alpha$$

$$P\left[\frac{0.0229}{0.0744}\frac{1}{4.96207} < \frac{\sigma_1^2}{\sigma_2^2} < \frac{0.0229}{0.0744}\frac{1}{0.25675}\right] = 1 - 2\alpha$$

$$P\left[0.0620 < \frac{\sigma_1^2}{\sigma_2^2} < 1.1988\right] = 0.95$$

The claimed ratio of population variances $\frac{\sigma_1^2}{\sigma_2^2} = \frac{0.05}{0.12} = 0.4167$ does fall within this interval.

The f values used in this problem could be obtained either from a table or from MatLab. The MatLab commands are given below.

```
>> f = icdf('f',0.01,15,9)

f =   0.256753377204427
```

and for the upper limit

```
f = icdf('f',0.99,15,9)

f =   4.962078356399956
```