Homework Assignment Number Twelve

Problem 1.
Consider the system of non-linear ordinary differential equation:

$$\frac{dy_1}{dx} = \frac{k_{11}y_1}{k_{12}y_2} + b_1 x$$
$$\frac{dy_2}{dx} = \sqrt{k_{21}y_1 + k_{22}y_2 + b_2 x}$$

with the initial conditions

$$y_1(x=0) = y_{1o} = 1.0$$
$$y_2(x=0) = y_{2o} = 0.1$$

where

$$k = \begin{bmatrix} -0.1 & 0.1 \\ 0.03 & 0.02 \end{bmatrix} \text{ and } b = \begin{bmatrix} -0.01 \\ 0.0 \end{bmatrix}$$

(a) Determine the behavior of $y_1(x)$ and $y_2(x)$ from $0 \le x \le 20$.
(b) Determine the values of $y_1(x)$ and $y_2(x)$ at $x = 10$.

Problem 2. (12 points)
We are testing a polymer membrane designed to catalytically filter microbes. The concentration of microbe A, $C_{A,0}$, on one side of the membrane, located at $z = 0$, is $C_{A,0}$. The concentration of microbe A, $C_{A,f}$, on the other side of the membrane, located at $z = L$, is $C_{A,f}$. Inside the polymer membrane, a chemical agent kills the microbe. The following equation describes the profile of the microbe concentration within the membrane,

$$0 = D\frac{d^2C_A}{dz^2} - k\sqrt{C_A}$$

(a) Is this ODE problem linear or nonlinear?
(b) Is this ODE problem an initial value problem or a boundary value problem?
(c) Convert this second order ODE into a system of two first order ODEs.

(d) For a membrane of thickness, L = 5 cm, and the following numerical values, $D = 1.0 \cdot 10^{-6} \frac{cm^2}{s}$, $k = 2.8 \cdot 10^{-9} \frac{\left(\frac{mol}{\ell}\right)^{1/2}}{s}$, $C_{A,0} = 1.0 \cdot 10^{-2} \frac{mol}{\ell}$ and $C_{A,f} = 2.2 \cdot 10^{-4} \frac{mol}{\ell}$, find the concentration gradient of the microbe at $z = 0$.
(e) Sketch the concentration profile.
(f) Verify that your discretization resolution was sufficient.