Homework Assignment Number Seven

For Problems 1, 2 and 3 below, find
(a) the determinant of A
(b) the reduced row echelon form of A
(c) the reduced row echelon form of the Augmented Ab matrix.
(d) the rank of A.
(e) the rank of the augmented Ab matrix.
(f) the inverse of A if it exists
(g) a solution to $\underline{\underline{A}}\underline{x} = \underline{b}$ if it exists.

Problem 1. (14 points)

$$\underline{\underline{A}} = \begin{bmatrix} 1 & 2 & -1 \\ 1 & 5 & 1 \\ 1 & 4 & 2 \end{bmatrix} \text{ and } \underline{b} = \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix}$$

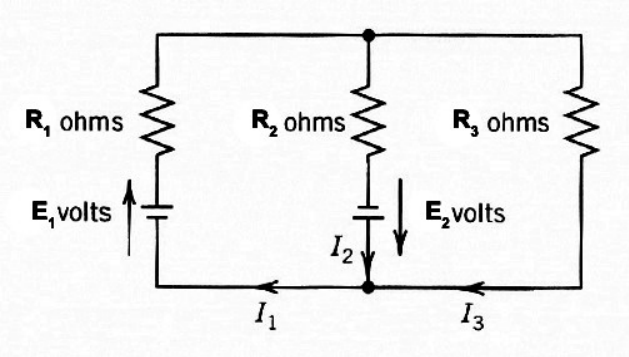
Problem 2. (14 points)

$$\underline{\underline{A}} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 1 & -1 \\ 1 & 3 & 2 & -2 \\ 1 & 4 & 3 & -3 \end{bmatrix} \text{ and } \underline{b} = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 2 \end{bmatrix}$$

Problem 3. (14 points)

$$\underline{\underline{A}} = \begin{bmatrix} 1 & 2 & -1 \\ 1 & 5 & 1 \\ 2 & 7 & 0 \end{bmatrix} \text{ and } \underline{b} = \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix}$$

Problem 4. (8 points)
Consider the following circuit.



Circuit analysis is performed by using Kirchoff's Current & Voltage laws. Kirchoff's current law states that the sum of the currents entering and leaving any node must sum to zero. So for the bottom node, Kirchoff's current law provides the following equation.

$$-i_1 + i_2 + i_3 = 0$$

where *i* denotes current. Kirchoff's voltage law states that in a loop the sum of the voltages is zero. The application of Kirchoff's voltage law to the right and left loops respectively yields

$$R_1 i_1 + R_2 i_2 = E_1 + E_2$$

$$-R_2 i_2 + R_3 i_3 = -E_2$$

where *R* denotes a resistance and *E* denotes an applied voltage. Consider an example where the following values are assigned: $R_1 = 15, R_2 = 15, R_3 = 25, E_1 = 50, E_2 = 80$

(a) Write this set of equations in matrix notation, $\underline{A}\underline{x} = \underline{b}$. Identify all three quantities, $\underline{A}$, $\underline{x}$ and $\underline{b}$.
(b) Calculate the determinant of $\underline{A}$.
(c) How many solutions are there to this problem?
(d) Calculate the currents.

Problem 5. (8 points)

For an ideal mixture, the volume of the mixture, V_{mix}, is given by the sum of the pure component molar volumes, V_i, weighted by the mole fraction, x_i. Similarly, the enthalpy of the mixture, H_{mix}, is given by the sum of the pure component enthalpies, H_i, weighted by the mole fraction. The cost of the mixture, C_{mix}, is also given by the sum of the pure component costs, C_i, weighted by the mole fraction. As a reminder, the sum of the mole fractions is unity.

$$V_{mix} = \sum_{i=1}^{n_c} x_i V_i \qquad H_{mix} = \sum_{i=1}^{n_c} x_i H_i \qquad C_{mix} = \sum_{i=1}^{n_c} x_i C_i \qquad 1 = \sum_{i=1}^{n_c} x_i$$

where n_c is the number of components. Now consider a four component ideal mixture ($n_c = 4$) with the following pure component properties and mixture properties.

component	A	B	C	D	mixture
molar volume (liter/mol)	11	14	17	8	12.8
enthalpy (kJ/mol)	49	74	45	63	57.1
cost ($/mol)	40.18	89.67	38.22	55.99	56.216

(a) Is this system of algebraic equations linear or nonlinear? (2 pts)
(b) Determine the composition of this mixture. Show reasoning and method. (6 pts)