Homework Assignment Number Four

Problem 1. (8 points)

You have a computer code that generates random integers in the range 20 to 40, inclusive.

(a) If the random variable, x, is the value of the random integer, what PDF describes the distribution of x?

(b) What is the probability that $25<x<35$?

(c) What is the mean value of the random integer?

(d) What is the variance of the integer?

Problem 2. (8 points)

According to Chemical Engineering Progress (Nov. 1990) approximately 30% of all pipework failures in chemical plants are caused by operator error.

(a) If x is a random variable that describes the number of pipework failures caused by operator error, what PDF will describe x?

(b) What is the probability that out of the next 20 pipework failures, at least 10 are due to operator error?

(c) What is the probability that no more than 4 out of 20 such failures are due to operator error?

(d) What is the probability that out of the next 20 pipework failures, exactly 5 are due to operator error?

Problem 3. (4 points)

According to genetics theory, a certain cross of guinea pigs will result in red, black, and white offspring in the ratio 8:4:8.

(a) If {x} is a set of random variables that describes the number of red, black, and white offspring, what PDF describes {x}?

(b) Find the probability that among 8 offspring 5 will be red, 2 black, and 1 white.

Problem 4. (4 points)

An urn contains 3 green balls, 2 blue balls, and 4 red balls. A random sample of 5 balls is selected.

(a) If {x} is a set of random variables that describes the number of balls of each color drawn, what PDF describes {x}?

(b) Find the probability that in the sample of 5 balls, both blue balls and at least 1 red ball are selected.

Problem 5. (4 points)

Population studies of biology and the environment often tag and release subjects in order to estimate size and degree of certain features in the population. 10 animals of a certain population thought to be near extinction are caught, tagged and released in a certain region. After a period of time, a random sample of 15 of the types of animals are caught. There are only 25 of the animals in the region.

(a) If x is a random variable that describes the number of animals caught both times, what PDF describes x?

(b) What is the probability that five of the animals caught in the second batch had been caught in the first batch?

Problem 6. (2 points)

Among 150 IRS employees in a large city, only 30 are women. (a) If 10 of the employees are chosen at random to provide free tax assistance to residents of the city, use the binomial approximation to the hypergeometric PDF to find the probability that at least three women were selected.

Problem 7. (6 points)

A scientist inoculates several mice, one at a time, with a disease germ until he finds 2 that have contracted the disease. If the probability of contracting the disease is 1/6.

(a) If x is a random variable that describes the number of mice which must be inoculated, what PDF describes x?

(b) What is the probability that 8 mice are required?

(c) What is the probability that between 8 and 10 mice, inclusive, are required?

Problem 8. (6 points)

The acceptance scheme for purchasing lots containing a large number of batteries is to test no more than 75 randomly selected batteries and to reject a lot if a single battery fails. Suppose the probability of a failure is 0.001.

(a) What is the probability that a lot is accepted?

(b) What is the probability that a lot is rejected on the 20^{th} attempt?

(c) What is the probability that it is rejected in 10 or less trials?

Problem 9. (6 points)

On average a certain intersection results in 3 traffic accidents per month. What is the probability that for any given month at this intersection

(a) exactly 5 accidents occur?

(b) less than 3 accidents occur?

(c) at least 2 accidents occur?