Homework Assignment Number Three

Problem 1. (38 points)

A chemical plant produces A thousands of liters of "A-plus Liquid Fungicide" and B thousand of liters of "B-Gone Liquid Insecticide" per month. The two processes share some raw materials and facilities so that the amount of A and B produced are not independent of each other. In fact the amount of B produced is related to the amount of A produced by

$$B = RM - \frac{A}{2} + 40$$

where RM is the amount of raw materials received at the plant in a given month (also in liters). The total amount of product in thousands of liters is given as

$$T(A,B) = B + A$$

The monthly production schedule for 2012 is as followed

Month	A (thousand of liters)	RM (thousands of liters)
Jan	50	120
Feb	50	120
Mar	60	110
Apr	70	120
May	80	130
Jun	90	130
Jul	100	130
Aug	100	130
Sep	90	130
Oct	80	120
Nov	70	110
Dec	60	100

In all problems: Put units with answers or you will not receive full credit.
In all relevant problems: Write down the formula before you use it.

(a.1) Is this problem continuous or discrete?
(a.2) What is the PDF?
(b) Find the average monthly production of A.
(c) Find the average monthly production of B. B is just a function of A and RM.
(d) Find the average monthly usage of RM.
(e) Find the mean of the total monthly production, T.
(f.1) Find the variance of the monthly production of A using the rigorous definition of the variance.
(f.2) Find the variance of the monthly production of A using the "mean of the squares minus the square of the mean" formula.
(g) Find the variance of the monthly usage of RM.

(h.1) Find the variance of the monthly production of B from tabulated values of B.

(h.2) Find the variance of the monthly production of B from the variances of A and RM and the formula for B given in this problem statement.

(i) Find the variance of the total monthly production, T.

(j,k,l,m) Find the standard deviations of A, B, RM and T.

(n) Find the covariance of A and B.

(o) Find the correlation coefficient of A and B

(p) Give a physical description of what the value and sign of the correlation coefficient means.

Problem 2. (16 points)

A chemical plant contains a jacketed vessel in which the following isomerization reaction takes place:

$$A \rightarrow B$$

The rate of the production of B, r_B [moles/hour], is given by

$$r_B = kC_A$$

where $C_A$is the concentration of A [moles/liter] and the reaction rate constant, k[liters/hour], is given as a function of the temperature, T [Kelvin], as

$$k = 20.0 \cdot e^{-\frac{10{,}000}{RT}}$$

where $R$is the gas constant [8.314 J/mole/K]. This (highly ideal) jacketed vessel keeps temperature perfectly constant at the set temperature of 400 K. The concentration in the tank is obtained from the mass balance

$$accumulation = in - out + generation$$

$$V\frac{dC_A}{dt} = QC_{A,in} - QC_A - kC_A$$

where Q is the volumetric flowrate [liters/hour], and has a numerical value of $Q = 9.0$ l/hour. V is the reactor volume, $V = 100.0$ liters. Rearrangement yields:

$$\left(\frac{V}{QC_{A,in} - QC_A - kC_A}\right)dC_A = dt$$

and where $C_{A,in}$is the inlet concentration of A, $C_{A,in} = 2.0$ mole/liter. We can integrate this equation to yield

$$\frac{V}{(Q+k)}ln\left(\frac{kC_{A,in}}{QC_{A,in} - QC_A - kC_A}\right) = t$$

We can rearrange this equation to give us C_A

$$C_A(t) = \frac{C_{A,in}}{(Q+k)}\left(Q + ke^{-\left(\frac{Q+k}{V}\right)t}\right)$$

(a) Plot $C_A$and C_B on one graph and plot r_B as functions of t for $0 \leq t \leq 24$ hour. Remember, $C_B = C_{A,in} - C_A$.

(b) Identify the random variable, x,the lower limit of the random variable, a, the upper limit of the random variable, b, the PDF, $f(x)$, and a function of the random variable, $h(x)$.

(c) What is the average concentration of reactant, C_A, during that first day of operation?

(d) What is the average rate of production, r_B, during that first day of operation?

(e) What is the average concentration of B, C_B, during that first day of operation?

(f) What is the variance of C_A during that first day of operation?

(g) What is the variance of r_B during that first day of operation?

(h) What is the variance of C_B during that first day of operation?

Problem 3. (2 points)

A private pilot wishes to insure his airplane for $200,000. The insurance company estimates that a total loss may occur with a probability of 0.002, a 50% loss with probability 0.01 and a 25% loss with a probability of 0.1. Ignoring all other partial losses, what premium should the insurance company charge each year to realize an average profit of $500?