Homework Assignment Number Two

Problem 1. (2 points)

Determine the value of c so that the following functions can serve as a PDF of the discrete random variable *X*.

$f(x) = c(x^2 + 4)$ where $x = 0,1,2,3;$

Problem 2. (8 points)

A shipment of 7 computer monitors contains 2 defective monitors. A business makes a random purchase of 3 monitors. The random variable x is the number of defective monitors purchased by the company.

(a) Find the probability distribution of X. (This means you need three numbers, f(x=0), f(x=1), and f(x=2) because the random variable, X = number of defective monitors purchased, has a range from 0 to 2.

(b) Plot the PDF

(c) Find the cumulative PDF, F(x).

(d) Plot the cumulative PDF.

Problem 3. (10 points)

A continuous random variable, X, that can assume values between x=2 and x=5 has a PDF given by

$$f(x) = \frac{2}{27}(1 + x)$$

(a) Find P(X<4).

(b) Find P(3<X<4).

(c) Plot the PDF.

(d) Determine the analytical expression for the cumulative PDF.

(e) Plot the cumulative PDF.

Problem 4. (8 points)

Consider a system of particles that sit in an electric field where the energy of interaction with the electric field is given by E(x) = 2477.572 + 4955.144x, where x is spatial position of the particles. The probability distribution of the particles is given by statistical mechanics to be f(x) = c*exp(-E(x)/(R*T)) for 0<x<1 and 0 otherwise, where R = 8.314 J/mol/K and T = 270.0 Kelvin.

(a) Find the value of c that makes this a legitimate PDF.

(b) Find the probability that a particles sits at x<0.25

(c) Find the probability that a particles sits at x>0.75

(d) Find the probability that a particles sits at 0.25<x<0.75

Problem 5. (4 points)

Let X denote the reaction time, in seconds, to a certain stimulant and Y denote the temperature (reduced units) at which a certain reaction starts to take place. Suppose that the random variables X and Y have the joint PDF,

$$f(x,y) = \begin{cases} cxy \text{ for } 0 < x < 1; 0 < y < 2.1 \\ 0 \qquad \text{elsewhere} \end{cases}$$

where c = 0.907029.

(a) Find $P\left(0 \le X \le {}^{1}/_{2} \text{ and } {}^{1}/_{4} \le Y \le {}^{1}/_{2}\right)$

(b) Find $P(X < Y)$.

Problem 6. (6 points)

Let X denote the number of times that a control machine malfunctions per day (choices: 1, 2, 3) and Y denote the number of times a technician is called. f(x,y) is given in tabular form.

f(x,y)	**x**	**1**	**2**	**3**
y	1	0.05	0.05	0.1
	2	0.05	0.1	0.35
	3	0.0	0.2	0.1

(a) Evaluate the marginal distribution of X.

(b) Evaluate the marginal distribution of Y.

(c) Find P(Y = 3|X = 2).