

Homework Assignment Number Two

Problem 1.

Determine the value of c so that the following functions can serve as a PDF of the discrete random variable X .

$$f(x) = c(x^2 + 4) \text{ where } x = 0, 1, 2, 3;$$

Problem 2.

A shipment of 7 computer monitors contains 2 defective monitors. A business makes a random purchase of 3 monitors. If x is the number of defective monitors purchased by the company, find the probability distribution of X . (This means you need three numbers, $f(x=0)$, $f(x=1)$, and $f(x=2)$ because the random variable, X = number of defective monitors purchased, has a range from 0 to 2. Also, find the cumulative PDF, $F(x)$. Plot the PDF and the cumulative PDF. These two plots must be turned into class on the day the homework is due.

Problem 3.

A continuous random variable, X , that can assume values between $x=2$ and $x=5$ has a PDF given by

$$f(x) = \frac{2}{27}(1+x)$$

Find (a) $P(X < 4)$ and find (b) $P(3 < X < 4)$. Plot the PDF and the cumulative PDF. These two plots must be turned into class on the day the homework is due.

Problem 4.

Consider a system of particles that sit in an electric field where the energy of interaction with the electric field is given by $E(x) = 2477.572 + 4955.144x$, where x is spatial position of the particles. The probability distribution of the particles is given by statistical mechanics to be $f(x) = c \cdot \exp(-E(x)/(R \cdot T))$ for $0 < x < 1$ and 0 otherwise, where $R = 8.314 \text{ J/mol/K}$ and $T = 270.0 \text{ Kelvin}$.

- (a) Find the value of c that makes this a legitimate PDF.
- (b) Find the probability that a particles sits at $x < 0.25$
- (c) Find the probability that a particles sits at $x > 0.75$
- (d) Find the probability that a particles sits at $0.25 < x < 0.75$

Problem 5.

Let X denote the reaction time, in seconds, to a certain stimulant and Y denote the temperature (reduced units) at which a certain reaction starts to take place. Suppose that the random variables X and Y have the joint PDF,

$$f(x, y) = \begin{cases} cxy & \text{for } 0 < x < 1; 0 < y < 2.1 \\ 0 & \text{elsewhere} \end{cases}$$

where $c = 0.907029$. Find (a) $P(0 \leq X \leq \frac{1}{2} \text{ and } \frac{1}{4} \leq Y \leq \frac{1}{2})$ and (b) $P(X < Y)$.

Problem 6.

Let X denote the number of times that a control machine malfunctions per day (choices: 1, 2, 3) and Y denote the number of times a technician is called. $f(x,y)$ is given in tabular form.

$f(x,y)$	x	1	2	3
y	1	0.05	0.05	0.1
	2	0.05	0.1	0.35
	3	0.0	0.2	0.1

- (a) Evaluate the marginal distribution of X .
- (b) Evaluate the marginal distribution of Y .
- (c) Find $P(Y = 3|X = 2)$.