Midterm Examination Number One
Administered: Monday, February 8, 1999

In all relevant problems: WRITE DOWN THE FORMULA YOU USE, BEFORE YOU USE IT.
ALL PROBLEMS ARE WORTH 2 POINTS.

For Problems (1.1) to (1.7) Consider laminar flow down a circular pipe. 200 periodic readings of the maximum velocity (in the center of the pipe) and the viscosity were taken. We know that for laminar flow in a circular pipe that the average velocity [m/s] is:

$\bar{v} = \frac{v_{max}}{2}$, the mass flow rate [kg/s] is

$\dot{m} = \rho A \bar{v} = \rho \frac{\pi}{4} D^2 \bar{v}$, and the Reynold's number is

$N_{Re} = \frac{D\bar{v}\rho}{\mu}$. (Thus all three of these equations are functions of random variables, v_{max} and μ.)

The density, $\rho = 1000.0$ kg/m^3, and the diameter, $D = 0.01$ m, are constant. The following data is then tabulated for those 200 samples. (Not all of the data is shown below.)

run	vmax	mu	Nre	vmax^2	mu^2	Nre^2	vmax*Nre	m-flow-Nre	mu-Nre
	[m/s]	[kg/m/s]		[m/s]^2	[kg/m/s]^2		[m/s]	[kg/s]	[kg/m/s]
1	1.0411E+01	1.1663E-03	8.9266E+04	1.0838E+02	1.3601E-06	7.9685E+09	9.2933E+05	7.2989E+04	1.0411E+02
2	9.1656E+00	8.4819E-04	1.0806E+05	8.4008E+01	7.1942E-07	1.1677E+10	9.9045E+05	7.7789E+04	9.1656E+01
3	8.3684E+00	8.2060E-04	1.0198E+05	7.0030E+01	6.7339E-07	1.0400E+10	8.5340E+05	6.7026E+04	8.3684E+01
	…	…	…	…	…	…	…	…	…
198	1.1792E+01	1.1855E-03	9.9473E+04	1.3906E+02	1.4054E-06	9.8949E+09	1.1730E+06	9.2129E+04	1.1792E+02
199	9.1816E+00	1.0085E-03	9.1046E+04	8.4301E+01	1.0170E-06	8.2893E+09	8.3594E+05	6.5655E+04	9.1816E+01
200	9.5214E+00	8.9588E-04	1.0628E+05	9.0658E+01	8.0261E-07	1.1295E+10	1.0119E+06	7.9477E+04	9.5214E+01
sum	1.9934E+03	2.0227E-01	1.9994E+07	2.0097E+04	2.0734E-04	2.0518E+12	2.0162E+08	1.5835E+07	1.9934E+04
avg	9.9671E+00	1.0113E-03	9.9968E+04	1.0049E+02	1.0367E-06	1.0259E+10	1.0081E+06	7.9175E+04	9.9671E+01

(1.1) Find the variance of the average velocity, $\sigma^2_{\bar{v}}$.

(1.2) Find the standard deviation of the viscosity, σ_μ.

(1.3) Find the mean of the mass-flow, $\mu_{\dot{m}}$.

(1.4) Find the variance of the Reynold's number, $\sigma^2_{N_{Re}}$.

(1.5) Find the covariance of the viscosity and Reynold's number, $\sigma_{\mu N_{Re}}$.

(1.6) Find the correlation coefficient, $\rho_{\mu N_{Re}}$.

(1.7) Explain the sign of $\rho_{\mu N_{Re}}$.

For problems (2.1) to (2.5) consider the Joint PDF $f(x, y)$ given by:

y	x→	1	2	3	4
2		1/12	3/24	1/24	1/6
4		1/24	1/12	1/24	1/24
6		1/6	1/12	1/12	1/24

(2.1) Is this PDF discrete or continuous?
(2.2) Show this PDF is a valid PDF.
(2.3) Find $P(X \geq 4, Y = 4)$
(2.4) Find the marginal distribution, h(y)
(2.5) Find $P(X \geq 4 \mid Y = 4)$

For questions (3.1) to (3.6):
In sampling a population for the presence of a disease, the population is of two types: **I**nfected and **U**ninfected. The results of the test are of two types: **P**ositive and **N**egative. In rare disease detection, a high probability for detecting a disease can still lead to more false positives than true positives. Consider a case where a disease infects 1 out of every 100,000 individuals. The probability for a positive test result given that the subject is infected is 0.99. The probability for a negative test result given that the subject is uninfected is 0.999.

(3.1) For testing a single person, define the complete sample space.

(3.2) What is the probability of a false negative test result (a negative test result given that the subject is infected)?

(3.3) What is the probability of being uninfected AND having a negative test result?

(3.4) What is the probability of testing positive?

(3.5) Determine rigorously whether testing positive and having the disease are independent.

(3.6) Determine the percentage of people who test positive and who are really uninfected.