Exam III
Take Home Exam Solutions
Assigned: Friday, November 12, 1999
Due: Monday, November 15, 1999 BEGINNING OF CLASS

This exam is to be completed on an individual basis. A student who submits work discovered to have been done in conjunction with other people, regardless of whether they are currently in ChE 301 or not, will fail the entire exam.

Problem (1)

Consider the set of 3 nonlinear algebraic equations (that happen to describe the steady-state operation of a 1-1 heat exchanger with counter current flow).
Energy balance on the shell side fluid:

$$q = \dot{m}_{shell} C_{p,shell} \left| T_{out,shell} - T_{in,shell} \right|$$

Energy balance on the tube side fluid:

$$q = \dot{m}_{tubel} C_{p,tube} \left| T_{out,tube} - T_{in,tubel} \right|$$

Energy balance for heat transfer:

$$q = UA \frac{\left(T_{shell,in} - T_{tube,out}\right) - \left(T_{shell,out} - T_{tube,in}\right)}{\ln\left(\frac{T_{shell,in} - T_{tube,out}}{T_{shell,out} - T_{tube,in}}\right)}$$

We are given the mass flow rates: $\dot{m}_{shell} = 20\frac{kg}{s}$, $\dot{m}_{tube} = 10\frac{kg}{s}$

We are given the heat capacities: $C_{p,shell} = 5600\frac{J}{kg \cdot K}$, $C_{p,tube} = 4200\frac{J}{kg \cdot K}$

We are given the inlet temperatures: $T_{shell,in} = 500K$, $T_{tube,in} = 200K$

We are given the overall heat transfer coefficient: $U = 1000\frac{J}{m^2 \cdot K}$

We are given the heat transfer area: $A = 10 \cdot Lm^2$ where L is the length of the heat exchanger in meters.

(a) If $L = 1$ meter, find the outlet temperatures, $T_{out,shell}$ and $T_{out,tube}$, and find the heat transferred, q.

(b) If $L = 2$ meters, find the outlet temperatures, $T_{out,shell}$ and $T_{out,tube}$, and find the heat transferred, q.

(c) Find the length, L, required to cool the shell fluid to 450 K ($T_{out,shell} = 450K$). What are the resulting values of $T_{out,tube}$ and q?

solution:

For part (a), we enter the equations in sysegninput as shown below:

```
function [f] = syseqninput(x0)
```

```
Tshellout = x0(1);
Ttubeout = x0(2);
q = x0(3);
mshell = 20;
mtube = 10;
cpshell = 5600;
cptube = 4200;
Tshellin = 500;
Ttubein = 200;
U = 1000;
L = 1;
A = 10*L;
lndeltaTnum = (Tshellin - Ttubeout) - (Tshellout - Ttubein);
lndeltaTden = log( (Tshellin - Ttubeout)/(Tshellout - Ttubein) );
lndeltaT = lndeltaTnum /lndeltaTden ;
f(1) = q - mshell*cpshell*abs(Tshellout - Tshellin);
f(2) = q - mtube*cptube*abs(Ttubeout - Ttubein);
f(3) = q - U*A*lndeltaT;
```

At the MATLAB prompt, we type:

```
» syseqn([450,260,100])
Attempting solution with MATLABs fsolve function
VARIABLE    INPUT     OUTPUT
    1   4.5000000e+002  4.7701853e+002
    2   2.6000000e+002  2.6128393e+002
    3   1.0000000e+002  2.5739249e+006
```

This tells us that
$T_{out,shell} = 477.0K$ and $T_{out,tube} = 261.3K$, and $q = 2{,}574{,}000J$.

(b) We change L from 1 to 2 in the above code and resolve

```
» syseqn(1,[450,300,2.5e+6])
Attempting solution with MATLABs fsolve function
VARIABLE    INPUT     OUTPUT
    1   4.5000000e+002  4.5986412e+002
    2   3.0000000e+002  3.0702902e+002
    3   2.5000000e+006  4.4952188e+006
```

$T_{out,shell} = 459.9K$ and $T_{out,tube} = 307.0K$, and $q = 4{,}495{,}000J$.

(c)

```
function [f] = syseqninput(x0)
Tshellout = 450;
Ttubeout = x0(2);
q = x0(3);
mshell = 20;
mtube = 10;
cpshell = 5600;
cptube = 4200;
Tshellin = 500;
Ttubein = 200;
U = 1000;
L = x0(1);
```

```
A = 10*L;
lndeltaTnum = (Tshellin - Ttubeout) - (Tshellout - Ttubein);
lndeltaTden = log( (Tshellin - Ttubeout)/(Tshellout - Ttubein) );
lndeltaT = lndeltaTnum /lndeltaTden ;
f(1) = q - mshell*cpshell*abs(Tshellout - Tshellin);
f(2) = q - mtube*cptube*abs(Ttubeout - Ttubein);
f(3) = q - U*A*lndeltaT;
```

In MATLAB
» syseqn([10,307,5.1e+6])
Attempting solution with MATLABs fsolve function
VARIABLE INPUT OUTPUT
1 1.0000000e+001 2.7247255e+000
2 3.0700000e+002 3.3333333e+002
3 5.1000000e+006 5.6000000e+006

$L = 2.724m$ and $T_{out,tube} = 333.3K$, and $q = 5{,}600{,}000J$.

Problem 2.
Given the data file on the exam portion of the website, titled file.xm3_f99.dat, determine the coefficients of the best fit models of the form

$$y = b_0 + b_1 x + b_2 x^2$$

and

$$y = b_0 + b_1 x + b_2 x^2 + b_3 x^3$$

Clearly label which coefficients are which. Report standard deviations for each parameter. Report measures of fit for both models. Which model would you recommend using for the data? Explain.

Solution:

For the quadratic model:

» regress(2,2,100,1,'file.xm3_f99.dat')
COMPLETED A NON-LINEAR REGRESSION ON THE DATA IN file.xm3_f99.dat
THE NUMBER OF PARAMETERS IS 3
The zeroth order parameter was used.
THE NUMBER OF DATA POINTS IS 100
PARAMETER VALUE STANDARD DEVIATION
1 -8.7238389e+000 3.0845393e+001 (this is for b0)
2 -3.1306335e+000 2.5047021e+000 (this is for b1)
3 2.4566253e+000 4.2484038e-002 (this is for b2)
MOF = 9.9843024e-001
F = 6.1695898e+004
degrees of freedom = 9.7000000e+001

» regress(2,3,100,1,'file.xm3_f99.dat')
COMPLETED A NON-LINEAR REGRESSION ON THE DATA IN file.xm3_f99.dat
THE NUMBER OF PARAMETERS IS 4

The zeroth order parameter was used.
THE NUMBER OF DATA POINTS IS 100
PARAMETER VALUE STANDARD DEVIATION
1 3.7257985e+001 4.9427951e+001 (this is for b0)
2 -1.1122068e+001 7.1711256e+000 (this is for b1)
3 2.7894226e+000 2.8310269e-001 (this is for b2)
4 -3.8424633e-003 3.2318351e-003 (this is for b3)
MOF = 9.9845302e-001
F = 6.1960366e+004
degrees of freedom = 9.6000000e+001

The cubic equation of state gives a marginally better MOF. However, it must give a better MOF because it has more parameters. The very slight increase in the MOF does not justify the use of an additional parameter. The quadratic model is the recommended model.

Problem 3.

Provide a solution to

$$\underline{\underline{A}}\,\underline{x} = \underline{b}$$

where

$$\underline{\underline{A}} = \begin{bmatrix} 1 & 1 & 2 & 1 \\ 1 & 2 & 2 & 1 \\ 3 & 5 & 6 & 3 \\ 1 & 3 & 2 & 1 \end{bmatrix} \text{ and } \underline{b} = \begin{bmatrix} 1 \\ 2 \\ 5 \\ 3 \end{bmatrix}$$

Solution:

In Matlab, enter matrix and vector

```
» A=[1 1 2 1
1 2 2 1
3 5 6 3
1 3 2 1];
» b = [1;2;5;3];
```

The determinant turns out to be 0.

```
» det(A)
ans = 0
```

The rank is 2. There are only 2 independent equations and 2 independent unknowns. There are an infinite number of solutions.

```
» rank(A)
ans =  2
```

Let's randomly choose $x_3 = 1$ and $x_4 = 1$. Our new 2x2 matrix and 2x1 vector become:

```
» A2 = [1 1
1 2]
» b2 = [1-2-1;2-2-1];
```

The determinant of the 2x2 matrix is non-zero so we can solve.

```
» det(A2)
ans = 1
```

Solve for x

```
» x=inv(A2)*b2
x =
    -3
     1
```

A complete solution to the problem is then:

$$\underline{x} = \begin{bmatrix} -3 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

Check this solution:

```
» x=[-3;1;1;1];
» A*x-b

ans =
     0
     0
     0
     0
```

So it checks.