Exam III
Administered: Wednesday, November 4, 2026
18 points

For each problem part: 0 points if not attempted or no work shown,
1 point for partial credit, if work is shown,
2 points for correct numerical value of solution

Problem 1 (10 points)

Consider the ternary Au-Pb-Sn alloy at the eutectic point. The compositions of the four phases present at the eutectic point are given in the table below. Additionally, the total composition of the alloy is given in the last row of the table.

phase	Au (at%)	Pb (at%)	Sn (at%)
α (Pb rich)	0.0	0.71	0.29
β (Sn rich)	0.0	0.02	0.98
γ ($AuSn_4$)	0.2	0.0	0.8
L (liquid)	0.03	0.251	0.719
T (total)	0.08	0.212	0.708

The phase fractions must sum to unity.

$$f_\alpha + f_\beta + f_\gamma + f_L = 1 \tag{1}$$

The mass balances on each component are written as follows

$$f_\alpha x_{Au}^\alpha + f_\beta x_{Au}^\beta + f_\gamma x_{Au}^\gamma + f_L x_{Au}^L = x_{Au}^T \tag{2}$$

$$f_\alpha x_{Pb}^\alpha + f_\beta x_{Pb}^\beta + f_\gamma x_{Pb}^\gamma + f_L x_{Pb}^L = x_{Pb}^T \tag{3}$$

$$f_\alpha x_{Sn}^\alpha + f_\beta x_{Sn}^\beta + f_\gamma x_{Sn}^\gamma + f_L x_{Sn}^L = x_{Sn}^T \tag{4}$$

In this set of equations, the phase fractions, f_α, f_β and f_γ are unknown. The phase fraction of the liquid, f_L, is measured to be $f_L = 0.1$

(a) Are these equations linear or non-linear?
(b) Rearrange the system of equations into the form of $\underline{Ax} = \underline{b}$. Identify each variable.
(c) Provide the determinant of the matrix.
(d) How many solutions are there to this system of equations?
(e) Determine the remaining set of phase fractions, f_α, f_β and f_γ.

Problem 2. (8 points)

The Hall-Petch equation states that the yield strength, σ_y, varies with grain size, d, according to

$$\sigma_y = \sigma_o + k_y \frac{1}{\sqrt{d}} \tag{7}$$

where σ_o and k_y are material-specific parameters.
[Materials Science and Engineering: An Introduction, 5th Edition, William D. Callister, John Wiley & Sons, Inc., New York, 2000, p. 167.]

For the σ_y vs d data given in the file, http://utkstair.org/clausius/docs/mse301/data/xm3p02_f26.txt, perform the following tasks. The grain size is given in mm and the yield strength in MPa.

(a) Identify all variables, $y = mx + b$, when equation (7) is linearized.
(b) Report the best value of σ_o and k_y.
(c) Report the standard deviations of σ_o and k_y.
(d) Report the measure of fit.