Exam I Solutions
Administered: Wednesday, September 21, 2021
24 points

For each problem part: 0 points if not attempted or no work shown,
1 point for partial credit, <u>if work is shown</u>,
2 points for correct numerical value of solution

Problem 1. (12 points)

Consider the data for the following **17** refractory ceramics given below. This data is available electronically on the course website in a spreadsheet file.

Material	Melting Point	Max Temp	Hardness	Density	Specific Heat	Thermal Expansion (Linear)	Thermal Conductivity
	°C	°C	Moh's Scale	g/cm3	J/kg °C	10-6 / °C	W/m °C
Alumina	2050	1950	9	3.96	1050	8	4
Beryllia	2550	2400	9	3	2180	7.5	29
Magnesia	2850	2400	6	3.6	1170	13.5	59
Thoria	3220	2700	7	9.7	290	9.5	3
Zirconia	2700	2400	6.5	5.6	590	7.5	3
Zircon	2500	1870	7.5	4.6	630	4.5	4
Spinel	2130	1900	8	3.6	1050	8.5	2
Mullite	1850	1800	8	2.8	840	5	4
Sillimanite	1800	1800	6.5	3.2	840	5	2
Silicon Carbide	2200	1400	9	3.2	840	4.5	13
Silicon Nitride	1900	1400	9	3.18	1050	2.9	9.5
Graphite	3600	3273	0.75	2.2	1600	2.2	147
Quartzite	1400	3000	7	2.65	1170	8.6	2.6
Boron Carbide	2350	540	9.3	2.5	2090	5.7	17.3
Boron Nitride	2721	650	2	2.1	1570	7.5	26
Titanium Carbide	3140	1500	9.5	6.5	1050	6.9	40
Tungsten Carbide	2780	1000	9.5	14.3	300	6.3	43.3

Answer the following questions for the materials in this table.

(a) Determine the mean melting temperature.
(b) Determine the mean specific heat.
(c) Determine the standard deviation of the melting temperature.
(d) Determine the standard deviation of the specific heat.
(e) Determine the correlation coefficient between the melting temperature and the specific heat.
(f) What is the physical significance of your answer to part (e)?

Solution:
(a) Determine the mean melting temperature.

$$\mu_{T_m} = \frac{\sum_{i=1}^{n} T_{m_i}}{n} = 2455.4\ K$$

The mean melting temperature of these 17 materials is $2455.4\ K$.

(b) Determine the mean specific heat.

$$\mu_{C_p} = \frac{\sum_{i=1}^{n} C_{p_i}}{n} = 1077.1 \frac{J}{kg\cdot K}$$

The mean specific heat of these 17 materials is $1077.1 \frac{J}{kg \cdot K}$.

(c) Determine the standard deviation of the melting temperature.

$$\sigma_{T_m}^2 = E[T_m^2] - E[T_m]^2 = 6{,}342{,}331.824 - (2455.4)^2 = 313{,}573.7578\ (K)^2$$

$$\sigma_{T_m} = \sqrt{\sigma_{T_m}^2} = 559.98\ K$$

The standard deviation of the melting temperature is $559.98\ K$.

(d) Determine the standard deviation of the specific heat.

$$\sigma_{C_p}^2 = E[C_p^2] - E[C_p]^2 = 1{,}431{,}123.529 - (1077.1)^2 = 271067.8201 \left(\frac{J}{kg \cdot K}\right)^2$$

$$\sigma_{C_p} = \sqrt{\sigma_{C_p}^2} = 520.64 \frac{J}{kg \cdot K}$$

The standard deviation of the specific heat is $520.64 \frac{J}{kg \cdot K}$.

(e) Determine the correlation coefficient between the melting temperature and the specific heat.

$$\sigma_{T_m \cdot C_p} = E[T_m \cdot C_p] - E[T_m]E[C_p] = 2{,}647{,}398.235 - 2455.4 \cdot 1077.1 = 2838.685 \frac{J}{kg}$$

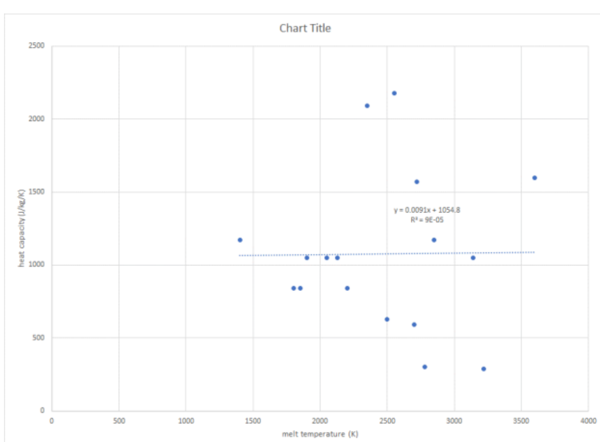
$$\rho_{T_m \cdot C_p} = \frac{\sigma_{T_m \cdot C_p}}{\sigma_{T_m} \cdot \sigma_{C_p}} = \frac{2838.685}{559.98 \cdot 520.64} = 0.009736624$$

The correlation coefficient of melting temperature and specific heat is 0.0097.

(f) What is the physical significance of your answer to part (e)?

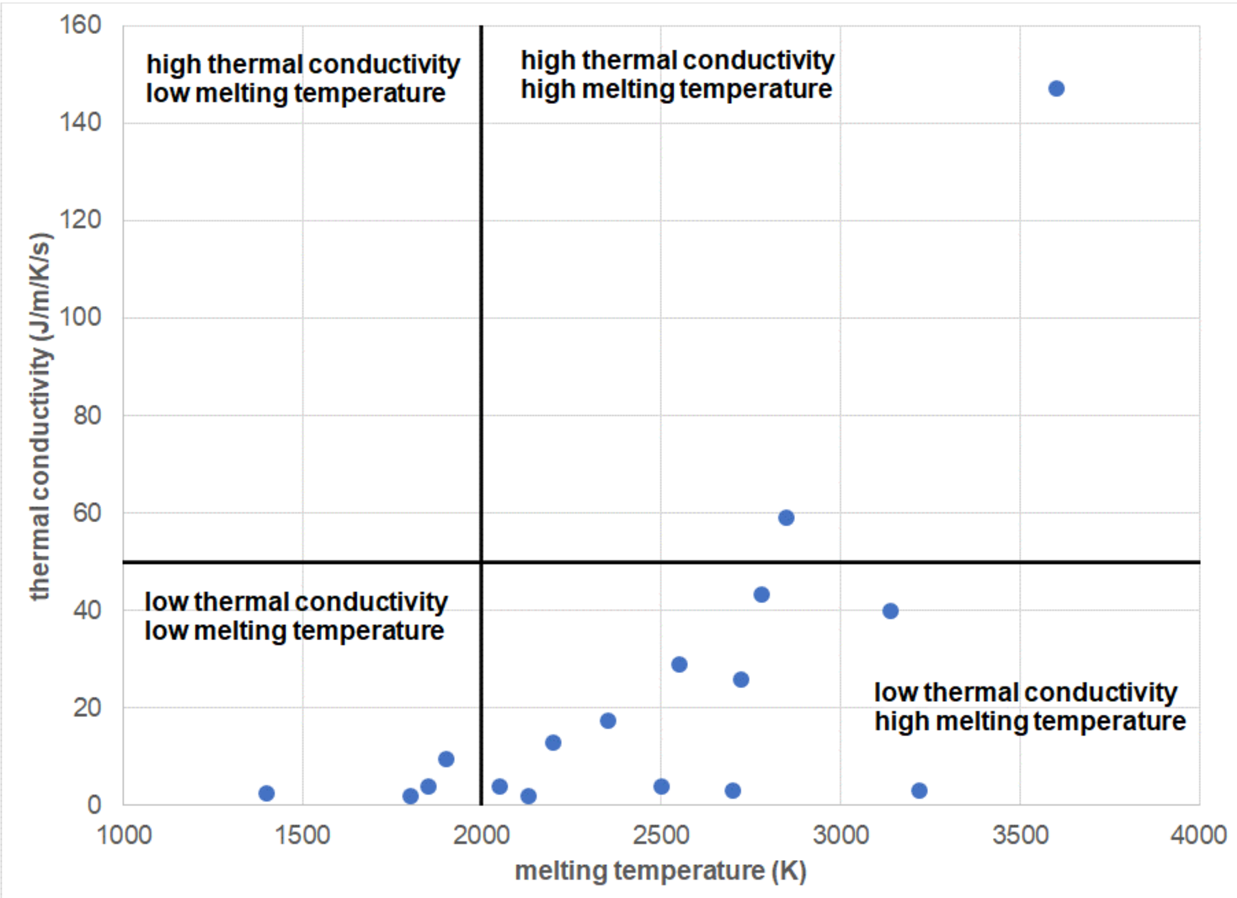
The value of the correlation coefficient is very close to zero. Therefore, there is no correlation between melting temperature and heat capacity. A change in the melting temperature does not statistically correspond to any change in the heat capacity.

A plot of the heat capacity vs the melting temperature is shown below (with a linear regression).



Problem 2. (12 points)

Consider the 17 ceramic materials in the table in Problem 1. We are evaluating these materials in terms of low or high thermal conductivity and low or high melting temperature. A plot of the thermal conductivity vs the melting temperature is shown below.



Using this information, answer the following questions.

(a) Draw a Venn Diagram of the sample space for this data.
(b) What is the probability that a material has high thermal conductivity and high melt temperature?
(c) What is the probability that a material has high thermal conductivity?
(d) What is the probability that a material has high melt temperature given that it had high thermal conductivity?
(e) What is the probability that a material has low thermal conductivity given that it had low melt temperature?
(f) Given this classification, prove that thermal conductivity and melt temperature are not independent of each other.

Solution:

There are a total of 17 materials.
From the plot we see:

$$P(\kappa_{lo} \cap T_{lo}) = \frac{4}{17}$$
$$P(\kappa_{hi} \cap T_{lo}) = \frac{0}{17}$$
$$P(\kappa_{lo} \cap T_{hi}) = \frac{11}{17}$$

$$P(\kappa_{hi} \cap T_{hi}) = \tfrac{2}{17}$$

(a) Draw a Venn Diagram of the sample space for this experiment.

$\kappa_{hi} \cap T_{lo}$	$\kappa_{hi} \cap T_{hi}$
$\kappa_{lo} \cap T_{lo}$	$\kappa_{lo} \cap T_{hi}$

(b) What is the probability that a material has high thermal conductivity and high melt temperature?

From the data given,

$$P(\kappa_{hi} \cap T_{hi}) = \tfrac{2}{17}$$

The probability that a material has high thermal conductivity and high melt temperature is $\tfrac{2}{17}$.

(c) What is the probability that a material has high thermal conductivity?

Consider the union probability rule.

$$P(\kappa_{hi}) = P(\kappa_{hi} \cap T_{hi}) + P(\kappa_{hi} \cap T_{lo}) - P[(\kappa_{hi} \cap T_{lo}) \cap (\kappa_{hi} \cap T_{hi})]$$
$$= \frac{2}{17} + 0 - 0 = \frac{2}{17}$$

The probability that a material has a high thermal conductivity can be computed from the union rule. The intersection of low melt temperature and high melt temperature is zero by definition of the categorization of these materials. The probability that a material has a high thermal conductivity is $\tfrac{2}{17}$.

(d) What is the probability that a material has high melt temperature given that it had high thermal conductivity?

Consider the conditional probability rule.

$$P(T_{hi}|\kappa_{hi}) = \tfrac{P(\kappa_{hi} \cap T_{hi})}{P(\kappa_{hi})} = \tfrac{2}{17} \Big/ \tfrac{2}{17} = 1$$

The probability that a material has high melt temperature given that it had high thermal conductivity is 1.0. All materials with high thermal conductivity also have high melt temperature.

(e) What is the probability that a material has low thermal conductivity given that it had low melt temperature?

Consider the conditional probability rule.

$$P(\kappa_{lo}|T_{lo}) = \tfrac{P(\kappa_{lo} \cap T_{lo})}{P(T_{lo})}$$

We can determine the denominator as

$$P(T_{lo}) = P(\kappa_{hi} \cap T_{lo}) + P(\kappa_{lo} \cap T_{lo}) - P[(\kappa_{hi} \cap T_{lo}) \cap (\kappa_{lo} \cap T_{lo})]$$
$$= 0 + \frac{4}{17} - 0 = \frac{4}{17}$$

$$P(\kappa_{lo}|T_{lo}) = \tfrac{P(\kappa_{lo} \cap T_{lo})}{P(T_{lo})} = \tfrac{4}{17} \Big/ \tfrac{4}{17} = 1$$

The probability that a material has low thermal conductivity given that it has low melt temperature is 1.0. All materials with low melt temperature also have low thermal conductivity.

(f) Given this classification, prove that thermal conductivity and melt temperature are not independent of each other.

There are a variety of ways to prove this.

If A & B are independent, then $P(A|B) = P(A)$ or $P(B|A) = P(B)$.

So for example, we can show that thermal conductivity and melt temperature are not independent of eac other through any of the following statements.

$$P(\kappa_{lo}|T_{lo}) = 1 \neq P(\kappa_{lo}) = \frac{15}{17}$$

$$P(T_{hi}|\kappa_{hi}) = 1 \neq P(T_{hi}) = \frac{13}{17}$$

Since these statements are not equalities, we know that the variables are not independent of each other.