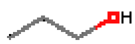
Exam II Solutions
Administered: Friday, October 11, 2019
24 points

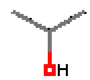
For each problem part: 0 points if not attempted or no work shown,
1 point for partial credit, if work is shown,
2 points for correct numerical value of solution

Problem 1. (16 points) Consider the following data for the enthalpy of fusion for two alcohols

1-Propanol
(C_3H_8O)



Isopropyl Alcohol
(C_3H_8O)



taken from the NIST Chemistry Webbook, http://webbook.nist.gov/chemistry/.

Enthalpy of fusion of 1-propanol

$\Delta_{fus}H$ (kJ/mol)	Temperature (K)	Reference
5.372	148.75	Counsell, Lees, et al., 1968, 2
5.4	148.7	van Miltenburg and van den Berg, 2004
5.37	148.8	Counsell, Lees, et al., 1968
5.192	147.0	Parks and Huffman, 1926, 2

Enthalpy of fusion of isopropyl alcohol

$\Delta_{fus}H$ (kJ/mol)	Temperature (K)	Reference
5.410	185.20	Andon, Counsell, et al., 1963
5.372	184.67	Kelley, 1929
5.41	185.2	Domalski and Hearing, 1996
5.301	184.6	Parks and Kelley, 1928
5.297	184.6	Parks and Kelley, 1925

Perform the following tasks.
(a) Determine the sample mean of the enthalpy of fusion of 1-propanol.
(b) Determine the sample mean of the enthalpy of fusion of isopropyl alcohol.
(c) Determine the sample variance of the enthalpy of fusion of 1-propanol.
(d) Determine the sample variance of the enthalpy of fusion of isopropyl alcohol.
(e) Identify the appropriate distribution to describe the difference of means in this case?
(f) Determine the lower limit of a 98% confidence interval on the difference of means of the enthalpy of fusion.
(g) Determine the upper limit of a 98% confidence interval on the difference of means of the enthalpy of fusion.
(h) Explain your findings in language a non-statistician can understand.

Solution:

(a) Determine the sample mean of the enthalpy of fusion of 1-propanol.

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{5} x_i = 5.3335\ \frac{kJ}{mol}$$

Based on this data set sample mean of the enthalpy of fusion of 1-propanol is 5.33 kJ/mol.

(b) Determine the sample mean of the enthalpy of fusion of isopropyl alcohol.

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{5} x_i = 5.358\ \frac{kJ}{mol}$$

Based on this data set sample mean of the enthalpy of fusion of isopropyl alcohol is 5.36 kJ/mol.

(c) Determine the sample variance of the enthalpy of fusion of 1-propanol.

$$s^2 = \frac{n\sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2}{n(n-1)} = 0.009086\left(\frac{kJ}{mol}\right)^2$$

Based on this data set the sample variance of the enthalpy of fusion of 1-propanol is 0.00909 (kJ/mol)^2.

(d) Determine the sample variance of the enthalpy of fusion of isopropyl alcohol.

$$s^2 = \frac{n\sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2}{n(n-1)} = 0.003144\left(\frac{kJ}{mol}\right)^2$$

Based on this data set the sample variance of the enthalpy of fusion of isopropyl alcohol is 0.00314 (kJ/mol)^2.

(e) Identify the appropriate distribution to describe the mean of the enthalpy of fusion of octane in this case.

In this case we do not know the true population variance so the appropriate distribution of the difference of sample means is the **t** distribution.

(f) Determine the lower limit of a 98% confidence interval on the difference of means of the enthalpy of fusion.
(g) Determine the upper limit of a 98% confidence interval on the difference of means of the enthalpy of fusion.

$$C.I. = 1 - 2\alpha = 0.98$$

$$\alpha = \frac{1 - C.I.}{2} = \frac{1 - 0.98}{2} = 0.01$$

$$v = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\left[\left(\frac{s_1^2}{n_1}\right)^2 \Big/ (n_1 - 1)\right] + \left[\left(\frac{s_2^2}{n_2}\right)^2 \Big/ (n_2 - 1)\right]} \quad \text{if } \sigma_1 \neq \sigma_2$$

$$v = 4.62 \approx 5$$

The limits on the t-distribution are given by

```
>> t = icdf('t',0.01,5)

t = -3.3649
```

and for the upper limit

```
>> t = icdf('t',0.99,5)

t =  3.3649
```

We next insert all of these numbers into the equation for the confidence interval on the difference of means.

$$P\left[\left(\overline{X}_1 - \overline{X}_2\right) + t_\alpha \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} < \left(\mu_1 - \mu_2\right) < \left(\overline{X}_1 - \overline{X}_2\right) + t_{1-\alpha} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}\right] = 1 - 2\alpha$$

$$P[\text{-}0.2057 < (\mu_1 - \mu_2) < \ 0.1567] = 0.98$$

(f) Translate your conclusions from (d) and (e) into a sentence that a non-statistician can understand.

We are 98% confidence that the difference between the enthalpy of fusion of 1-propanol and isopropyl alcohol lies within the range from -0.2057 to 0.1567 kJ/mol. While the sample mean of the enthalpy of fusion is larger for isopropyl alcohol than for 1-propanol, this confidence interval includes both positive and negative numbers,

Problem 2. (8 points)
Consider a battery with a distribution of lifetimes described by the normal distribution with a population mean of 5.0 years and a population variance of 1.0 years.
(a) What is the probability that a battery lasts at least 7.0 years?
(b) What is the probability that a device with a redundant power system running on four batteries has no batteries working in 7.0 years?
(c) What is the probability that a device with a redundant power system running on four batteries has all batteries working in 7.0 years?
(d) What is the probability that a device with a redundant power system running on four batteries has at least one battery still working in 7.0 years?

Solution:

(a) What is the probability that a battery lasts at least 7.0 years?

$$P(x \geq 7) = 1 - P(x \leq 7)$$

We use the cdf function in MATLAB:

```
>> p = 1 - cdf('normal',7,5,1.0)

p =   0.022750131948179
```

There is a 2.28% chance that a battery lasts at least 7 years.

(b) What is the probability that a device with a redundant power system running on four batteries has no batteries working in 7.0 years?

This problem requires the binomial distribution with x=0, n=4 and p, the probability of a single battery working after 7 years, defined in part a. We use the pdf function in Matlab because the binomial distribution is discrete and a discrete pdf gives the probability that x = a given value.

$$P(x = 0) = \mathrm{pdf}('\mathrm{binomial}', \mathrm{x} = 0, \mathrm{n} = 4, \mathrm{p} = 0.022750131948179)$$

```
>> p = pdf('binomial',0,4,0.022750131948179)

p =   0.912058052099395
```

There is a 91.2% chance that the device has zero batteries functioning after 7 years.

(c) What is the probability that a device with a redundant power system running on four batteries has all batteries working in 7.0 years?

This problem requires the binomial distribution with x=4, n=4 and p, the probability of a single battery working after 7 years, defined in part a.

$$P(x = 4) = \mathrm{pdf}('\mathrm{binomial}', \mathrm{x} = 4, \mathrm{n} = 4, \mathrm{p} = 0.022750131948179)$$

```
>> p = pdf('binomial',4,4,0.022750131948179)

p =     2.678771559803907e-07
```

There is a 0.0000268% chance that the device has all four batteries functioning after 7 years.

(d) What is the probability that a device with a redundant power system running on four batteries has at least one battery still working in 7.0 years?

This problem requires the binomial distribution with $x \geq 1$, $n=4$ and p, the probability of a single battery working after 7 years, defined in part a.

$$P(x \geq 1) = 1 - P(x \leq 0) = 1 - P(x = 0)$$

```
>> p = 1 - pdf('binomial',0,4,0.022750131948179)

p =   0.087941947900605
```

There is a 8.79% chance that the device has at least one battery functioning after 7 years.