Exam I Solutions
Administered: Wednesday, September 18, 2018
32 points

For each problem part: 0 points if not attempted or no work shown,
1 point for partial credit, if work is shown,
2 points for correct numerical value of solution

Problem 1. (12 points)

Consider the data for the following **12** polymers given below.

Polymer	T_{glass} (K)	T_{melt} (K)	T_{glass}^2 (K^2)	T_{melt}^2 (K^2)	$T_{glass}T_{melt}$ (K^2)
Nylon-6	326	496	106276	246016	161696
Nylon-6,6	324	538	104976	289444	174312
Polycarbonate	418	500	174724	250000	209000
Polyethylene	393	391	154449	152881	153663
Poly(ethylene terephthalate)	346	540	119716	291600	186840
Poly(methyl methacrylate)	388	433	150544	187489	168004
polyoxymethylene	216	460	46656	211600	99360
Polypropylene	259	450	67081	202500	116550
Polystyrene	373	513	139129	263169	191349
Polytetrafluoroethylene	200	602	40000	362404	120400
Poly(vinyl chloride)	354	546	125316	298116	193284
Silicone rubber	150	230	22500	52900	34500
sum	3747	5699	1251367	2808119	1808958

Answer the following questions.

(a) Determine the mean glass transition temperature.
(b) Determine the mean melting temperature.
(c) Determine the standard deviation of the glass transition temperature.
(d) Determine the standard deviation of the melting temperature.
(e) Determine the correlation coefficient between the glass transition temperature and the melting temperature.
(f) What is the physical significance of your answer to part (e)?

Solution:
(a) Determine the mean glass transition temperature.

$$\mu_{Tg} = \frac{\sum_{i=1}^{n} T_{g,i}}{n} = \frac{3747}{12} = 312.25\ K$$

The mean glass transition temperature of these 12 polymers is 312.25 K.

(b) Determine the mean melting temperature.

$$\mu_{T_m} = \frac{\sum_{i=1}^{n} T_{m,i}}{n} = \frac{5699}{12} = 474.92\ K$$

The mean melting temperature of these 12 polymers is 474.92 K.

(c) Determine the standard deviation of the glass transition temperature.

$$\sigma_{T_g}^2 = E\left[T_g^2\right] - E\left[T_g\right]^2 = \frac{1251367}{12} - (312.25)^2 = 6780.52\ K^2$$

$$\sigma_{T_g}^2 = \sqrt{\sigma_{T_g}^2} = 82.34\ K$$

The standard deviation of the glass transition temperature is 82.34 K.

(d) Determine the standard deviation of the melting temperature.

$$\sigma_{T_m}^2 = E[T_m^2] - E\left[T_m\right]^2 = \frac{2808119}{12} - (474.92)^2 = 8464.08\ K^2$$

$$\sigma_{T_m}^2 = \sqrt{\sigma_{T_m}^2} = 92.00\ K$$

The standard deviation of the glass transition temperature is 92.00 K.

(e) Determine the correlation coefficient between the glass transition temperature and the melting temperature.

$$\sigma_{T_g \cdot T_m} = E\left[T_g \cdot T_m\right] - E\left[T_g\right]E[T_m] = \frac{1808958}{12} - 312.25 \cdot 474.92 = 2453.77$$

$$\rho_{T_g \cdot T_m} = \frac{\sigma_{T_g \cdot T_m}}{\sigma_{T_g} \cdot \sigma_{T_m}} = \frac{2453.77}{82.34 \cdot 92.00} = 0.3239$$

The correlation coefficient of glass transition temperature and the melt temperature is 0.3239.

(f) What is the physical significance of your answer to part (e)?

A positive correlation coefficient indicates that, from a statistical point of view, as the glass transition temperature increases, the melting temperature increases (or equivalently as the glass transition temperature decreases, the melting temperature decreases).

Problem 2. (10 points)

A study is performed involving 3D printed tensile test specimens manufactured from acrylonitrile butadiene styrene (ABS) or polylactic acid (PLA) anode. Each specimen is tested and is categorized as either Pass (P) or Fail (F).

During this study the following information was collected.

# of ABS specimens that passed the test = 3
# of ABS specimens that failed the test = 17
# of PLA specimens that passed the test = 18
# of PLA specimens that failed the test = 2

Using this information, answer the following questions.

(a) Draw a Venn Diagram of the sample space for this experiment.
(b) What is the probability that a specimen in this study contained ABS?
(c) What is the probability that a specimen passed the test given that it contained ABS?
(d) What is the probability that a specimen passed the test given that it contained PLA?
(e) What is the probability that the specimen contained PLA given that the test failed?

Solution:

P = pass
F = fail

There are a total of 3+17+18+2 = 40 batteries
We are given:

$$P(ABS \cap P) = \frac{3}{40}$$
$$P(ABS \cap F) = \frac{17}{40}$$
$$P(PLA \cap P) = \frac{18}{40}$$
$$P(PLA \cap F) = \frac{2}{40}$$

(a) Draw a Venn Diagram of the sample space for this experiment.

$ABS \cap P$	$ABS \cap F$
$PLA \cap P$	$PLA \cap F$

(b) What is the probability that a specimen in this study contained ABS?

We can use the union rule.

$$P(ABS) = P(ABS \cap P) + P(ABS \cap F) - P[(ABS \cap P) \cap (ABS \cap F)] = \frac{3}{40} + \frac{17}{40} - 0 = \frac{20}{40} = \frac{1}{2}$$

The probability that a specimen contained ABS in this study is 0.5.

(c) What is the probability that a specimen passed the test given that it contained ABS?

Consider the conditional probability rule.

$$P(P|ABS) = \frac{P(ABS \cap P)}{P(ABS)} = \frac{\frac{3}{40}}{\frac{20}{40}} = \frac{3}{20} = 0.15$$

The probability that a specimen passed the test given that it contained ABS is 0.15.

(d) What is the probability that a specimen passed the test given that it contained PLA?

$$P(P|PLA) = \frac{P(PLA \cap P)}{P(PLA)} = \frac{\frac{18}{40}}{\frac{20}{40}} = \frac{18}{20} = 0.9$$

The probability that a specimen passed the test given that it contained PLA is 0.9.

(e) What is the probability that the specimen contained PLA given that the test failed?

$$P(F) = P(F \cap ABS) + P(F \cap PLA) - P[(F \cap ABS) \cap (F \cap PLA)] = \frac{17}{40} + \frac{2}{40} - 0 = \frac{19}{40}$$

Use the conditional probability rule

$$P(PLA|F) = \frac{P(PLA \cap F)}{P(F)} = \frac{\frac{2}{40}}{\frac{19}{40}} = \frac{2}{19}$$

The probability that a specimen contained PLA given that the test failed the test is 0.105.

Problem 3. (10 points)

Consider the following probability density function.

$$f(x) = \begin{cases} c(x^2+1) & for\ 0 < x < 2 \\ 0 & otherwise \end{cases}$$

(a) Is the PDF continuous or discrete?
(b) Find the value of *c* that normalizes this PDF.
(c) Find the probability that x is less than 1.0.
(d) Find the probability that x is greater than 1.0.
(e) Find the average value of the random variable x.

Solution:

(a) Is the PDF continuous or discrete?

This PDF is continuous.

(b) Find the value of c that normalizes this PDF.

$$1=\int_0^2 f(x)\,dx = c\int_0^2 (x^2+1)\,dx = c\left[\frac{x^3}{3}+x\right]_0^2 = c\,\frac{14}{3}$$

The value of *c* that normalizes this PDF is $c = \frac{3}{14} \sim 0.214$.

(c) Find the probability that x is less than 1.0.

$$P(x<1.0) = \int_0^{1.0} f(x)\,dx = \frac{3}{14}\int_0^1 (x^2+1)\,dx = \frac{3}{14}\left[\frac{x^3}{3}+x\right]_0^1 = \frac{3}{14}\frac{4}{3} = \frac{2}{7} \sim 0.2857$$

(d) Find the probability that x is greater than 1.0.

$$P(x>1.0) = 1 - P(x<1.0) = 1 - \frac{2}{7} = \frac{5}{7} \sim 0.7143$$

(e) Find the average value of the random variable x.

$$E[x] = \int_0^2 xf(x)\,dx = \frac{3}{14}\int_0^2 x(x^2+1)\,dx = \frac{3}{14}\left[\frac{x^4}{4}+\frac{x^2}{2}\right]_0^2 = \frac{3}{14}\left[\frac{16}{4}+\frac{4}{2}\right] = \frac{3}{14}[6] = \frac{18}{14} = \frac{9}{7} \sim 1.2857$$