Final Exam
Administered: Monday, December 9, 2019
5:00 PM – 7:00 PM
28 points

Problem 1. (6 points)

The error function, erf(x) is defined as

$$erf(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt \quad \text{for } x > 0$$

(a) Evaluate the error function for x = 1.7 using the intrinsic gamma function in Matlab, `erf`. You likely will need to use the `format long` statement in MatLab to get enough digits to display.
(b) How many intervals so you need in the second-order Simpson's method to obtain this result to six significant digits?
(c) In statistics, for nonnegative values of x, the error function has the following interpretation: for a random variable Y that is normally distributed with mean 0 and variance 1/2, erf(x) is the probability of Y falling in the range [−x, x]. Knowing this, show how can you use the Matlab `cdf` command to evaluate this integral.

Problem 2. (14 points)
A cylindrical titanium rod, of diameter, *d*, and length *L*, is horizontally suspended between two heat reservoirs, which maintain the temperature at one end (z=0) at 500 K and at the other end (z=l) at 1000 K. Between them a fan flows on the rod to conduct heat away. The steady state heat equation describing this set up is given below as

$$0 = \frac{k_c}{\rho C_p}\frac{d^2T}{dz^2} - \frac{h}{\rho C_p}\frac{A}{V}(T - T_{surr})$$

where

- k_c is the thermal conductivity, $k_c = 21.9 \frac{W}{m \cdot K}$
- ρ is the mass density, $\rho = 4506.0 \frac{kg}{m^3}$
- C_p is the specific heat capacity, $C_p = 523.5 \frac{J}{kg \cdot K}$
- d is the diameter of the rod, $d = 0.05\, m$
- l is the length of the rod, $l = 0.5\, m$
- A is the surface area of the rod, $A = \pi d l$
- V is the volume of the rod, $V = \frac{\pi}{4} d^2 l$
- A/V is the surface area to volume ratio of the rod, $A/V = \frac{4}{d}$
- T_{surr} is the surrounding temperature, $T_{surr} = 300K$
- h is an emprical heat transfer coefficient, $h = 40.0 \frac{W}{m^2 \cdot K}$

Problem 2. (continued)

Answer the following questions and perform the following tasks.

(a) Is this ODE problem linear or nonlinear?
(b) Is this ODE problem an initial value problem or a boundary value problem?
(c) Convert this second order ODE into a system of two first order ODEs.
(d) Find the initial temperature gradient at $z = 0$.
(e) Sketch the temperature profile.
(f) Verify that your discretization resolution was sufficient.
(g) What is the temperature in the middle of the rod at steady state?

Problem 3. (8 points)

Consider the isomerization reaction:

$$A \rightarrow B$$

The reaction rate is given by

$$rate = C_A k_o e^{-\frac{E_a}{RT}} \quad \text{[moles/m}^3\text{/sec]}$$

where

concentration of A: C_A [moles/m^3]
prefactor: k_o [1/sec]
activation energy for reaction: E_a [Joules/mole]
constant: $R = 8.314$ [Joules/mole/K]
temperature: T [K]

The reaction is measured at a constant concentration of A, C_A = 1000 mol/m^3, over a range of temperatures. The rate is recorded. The rate as a function of temperature is given in tabular form in the file "xm4p03_f19.txt" on the exam portion of the course website.

(a) Linearize this equation in the unknown reaction parameters.
(b) Perform a linear regression and report the measure of fit.
(c) Determine the rate constants, k_o and E_a, from experimental data.