

Final Exam  
Administered: Monday, December 9, 2019  
5:00 PM – 7:00 PM  
28 points

**Problem 1. (6 points)**

The error function,  $\text{erf}(x)$  is defined as

$$\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt \quad \text{for } x > 0$$

- (a) Evaluate the error function for  $x = 1.7$  using the intrinsic gamma function in Matlab, `erf`. You likely will need to use the `format long` statement in MatLab to get enough digits to display.
- (b) How many intervals so you need in the second-order Simpson's method to obtain this result to six significant digits?
- (c) In statistics, for nonnegative values of  $x$ , the error function has the following interpretation: for a random variable  $Y$  that is normally distributed with mean 0 and variance 1/2,  $\text{erf}(x)$  is the probability of  $Y$  falling in the range  $[-x, x]$ . Knowing this, show how can you use the Matlab `cdf` command to evaluate this integral.

**Problem 2. (14 points)**

A cylindrical titanium rod, of diameter,  $d$ , and length  $L$ , is horizontally suspended between two heat reservoirs, which maintain the temperature at one end ( $z=0$ ) at 500 K and at the other end ( $z=l$ ) at 1000 K. Between them a fan flows on the rod to conduct heat away. The steady state heat equation describing this set up is given below as

$$0 = \frac{k_c}{\rho C_p} \frac{d^2 T}{dz^2} - \frac{h}{\rho C_p} \frac{A}{V} (T - T_{surr})$$

where

- $k_c$  is the thermal conductivity,  $k_c = 21.9 \frac{W}{m \cdot K}$
- $\rho$  is the mass density,  $\rho = 4506.0 \frac{kg}{m^3}$
- $C_p$  is the specific heat capacity,  $C_p = 523.5 \frac{J}{kg \cdot K}$
- $d$  is the diameter of the rod,  $d = 0.05 \text{ m}$
- $l$  is the length of the rod,  $l = 0.5 \text{ m}$
- $A$  is the surface area of the rod,  $A = \pi d l$
- $V$  is the volume of the rod,  $V = \frac{\pi}{4} d^2 l$
- $A/V$  is the surface area to volume ratio of the rod,  $A/V = \frac{4}{d}$
- $T_{surr}$  is the surrounding temperature,  $T_{surr} = 300K$
- $h$  is an emprical heat transfer coefficient,  $h = 40.0 \frac{W}{m^2 \cdot K}$

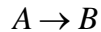
**Problem 2. (continued)**

Answer the following questions and perform the following tasks.

- (a) Is this ODE problem linear or nonlinear?
- (b) Is this ODE problem an initial value problem or a boundary value problem?
- (c) Convert this second order ODE into a system of two first order ODEs.
- (d) Find the initial temperature gradient at  $z = 0$ .
- (e) Sketch the temperature profile.
- (f) Verify that your discretization resolution was sufficient.
- (g) What is the temperature in the middle of the rod at steady state?

**Problem 3. (8 points)**

Consider the isomerization reaction:



The reaction rate is given by

$$rate = C_A k_o e^{-\frac{E_a}{RT}} \quad [\text{moles/m}^3/\text{sec}]$$

where

concentration of A:  $C_A$  [moles/m<sup>3</sup>]

prefactor:  $k_o$  [1/sec]

activation energy for reaction:  $E_a$  [Joules/mole]

constant:  $R = 8.314$  [Joules/mole/K]

temperature:  $T$  [K]

The reaction is measured at a constant concentration of A,  $C_A = 1000 \text{ mol/m}^3$ , over a range of temperatures. The rate is recorded. The rate as a function of temperature is given in tabular form in the file “xm4p03\_f19.txt” on the exam portion of the course website.

- (a) Linearize this equation in the unknown reaction parameters.
- (b) Perform a linear regression and report the measure of fit.
- (c) Determine the rate constants,  $k_o$  and  $E_a$ , from experimental data.