

Exam I
Administered: Wednesday, September 19, 2018
34 points

For each problem part: 0 points if not attempted or no work shown,
 1 point for partial credit, if work is shown,
 2 points for correct numerical value of solution

Problem 1. (14 points)

Consider the data for the following 24 metals given below, taken from <https://www.tibtech.com/conductivite.php>

	Electric conductivity (10^6 S/m)	Electric resistivity (10^{-8} Ω .m)	Thermal conductivity (W/m.K)	Thermal expansion 10^{-6} (K $^{-1}$)	Density (g/cm 3)	Melt/decompose temperature (°C)
Silver	62.1	1.6	420	19.1	10.5	961
Copper	58.7	1.7	386	17	8.9	1083
Gold	44.2	2.3	317	14.1	19.4	1064
Aluminium	36.9	2.7	237	23.5	2.7	660
Molybdenum	18.7	5.34	138	4.8	10.2	2623
Zinc	16.6	6	116	31	7.1	419
Lithium	10.8	9.3	84.7	56	0.54	181
Brass	15.9	6.3	150	20	8.5	900
Nickel	14.3	7	91	13.3	8.8	1455
Steel	10.1	9.9	80	12.1	7.9	1528
Palladium	9.5	10.5	72	11	12	1555
Platinum	9.3	10.8	107	9	21.4	1772
Tungsten	8.9	11.2	174	4.5	19.3	3422
Tin	8.7	11.5	67	23.5	7.3	232
Bronze 67Cu33Sn	7.4	13.5	85	17	8.8	1040
Carbone steel	5.9	16.9	54	12	7.7	1400
Carbone	5.9	16.9	129	0.2	1.8	2500
Lead	4.7	21.3	35	29	11.3	327
Titanium	2.4	41.7	21	8.9	4.5	1668
Stainless steel 316L EN1.4404	1.32	76	15	16.5	7.9	1535
Stainless steel 304 EN1.4301	1.37	73	16.3	16.5	7.9	1450
Stainless steel 310 EN1.4841	1.28	78	14.2	17	7.75	2650
Mercury	1.1	90.9	8	61	13.5	-39
FeCrAl	0.74	134	16	11.1	7.2	-1440

Based on this data, the following data analysis has been performed.

property	symbol	mean
Electric conductivity (10^6 S/m)	sigma	14.86708333
Electric resistivity (10^{-8} W.m)	r	27.43083333
Thermal conductivity (W/m.K)	k	118.05
Thermal expansion $10^{-6}(K^{-1})$	beta	18.67083333
Density (g/cm^3)	rho	9.287083333
Melt/decompose temperature ($^{\circ}C$)	Tmelt	1326.083333

mean of the square (diagonal) or product (off-diagonal)						
	sigma	r	k	beta	rho	Tmelt
sigma	517.0598	99.97283	3634.471	268.4808	148.8138	17058.8
r		1966.424	1003.868	583.3884	238.24	21767.89
k			26656.98	1985.823	1211.977	151556.9
beta				543.9029	160.8229	14499.41
rho					111.4406	12403.41
Tmelt						2461391

variance (diagonal) and covariance (off-diagonal)						
	sigma	r	k	beta	rho	Tmelt
sigma	(a)	(c)	1879.411938	-9.100043403	10.74194983	-872.1414236
r		1213.973366	-2234.341958	71.23189931	-16.51243507	-11315.98257
k			12721.17333	-218.2689583	115.6372292	9178.808333
beta				195.3028993	-12.5746684	-8019.168403
rho					25.19067066	1202.413576
Tmelt						1006753.91

correlation coefficient						
	sigma	r	k	beta	rho	Tmelt
sigma	1	-0.5135208	0.96848	-0.03785	0.124393	-0.050519
r		1	-0.56857	0.14629	-0.09442	-0.323688
k			1	-0.13848	0.204275	0.0811075
beta				1	-0.17928	(d)
rho					1	0.2387659
Tmelt						1

Answer the following questions.

- Determine the variance of the electric conductivity. Include units.
- Determine the standard deviation of the thermal conductivity. Include units.
- Determine the covariance of electric conductivity and electric resistivity. Include units.
- Determine the correlation coefficient of thermal expansion coefficient and melt temperature.
- Are the electrical conductivity and thermal conductivity independent variables? Justify answer.
- What property is most strongly correlated with density?
- Based on your understanding of physics, should the electrical conductivity be positively or negatively correlated with electrical resistivity? Justify answer.

Problem 2. (10 points)

A study is performed involving lithium ion batteries manufactured with a flake graphite (G) or lignin-based (L) anode. Each battery is evaluated for one hundred cycles and is categorized as either Pass (P) or Fail (F).

During this study the following information was collected.

of batteries with a flake graphite anode that passed the test = 19
of batteries with a flake graphite anode that failed the test = 6
of batteries with a lignin-based anode that passed the test = 13
of batteries with a lignin-based anode that failed the test = 2

Using this information, answer the following questions.

- (a) Draw a Venn Diagram of the sample space for this experiment.
- (b) What is the probability that a battery in this study contained a flake graphite anode?
- (c) What is the probability that a battery passed the test given that it contained a flake graphite anode?
- (d) What is the probability that a battery passed the test given that it contained a lignin-based anode?
- (e) What is the probability that the battery contained the lignin-based anode given that the test failed?

Problem 3. (10 points)

Consider the following probability density function.

$$f(x) = \begin{cases} cx^4 & \text{for } 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

- (a) Is the PDF continuous or discrete?
- (b) Find the value of c that normalizes this PDF.
- (c) Find the probability is less than 0.2.
- (d) Find the probability is greater than 0.2.
- (e) Find the average value of the random variable x.