Final Exam
Administered: Friday, December 8, 2017
40 points

For each problem part: 0 points if not attempted or no work shown,
1 point for partial credit, if work is shown,
2 points for correct numerical value of solution

Problem 1. (12 points)
A cylindrical aluminum rod, of diameter, *d*, and length *L*, is horizontally suspended between two heat reservoirs, which maintain the temperature at one end (z=0) at 400 K and at the other end (z=l) at 800 K. Between them a fan flows on the rod to conduct heat away. The steady state heat equation describing this set up is given below as

$$0 = \frac{k_c}{\rho C_p}\frac{d^2T}{dz^2} - \frac{h}{\rho C_p}\frac{A}{V}\left(T - T_{surr}\right)$$

where

- k_c is the thermal conductivity, $k_c = 237.0 \frac{W}{m \cdot K}$
- ρ is the mass density, $\rho = 2700.0 \frac{kg}{m^3}$
- C_p is the specific heat capacity, $C_p = 896.9 \frac{J}{kg \cdot K}$
- d is the diameter of the rod, $d = 0.025\, m$
- l is the length of the rod, $l = 0.5\, m$
- A is the surface area of the rod, $A = \pi d l$
- V is the volume of the rod, $V = \frac{\pi}{4} d^2 l$
- A/V is the surface area to volume ratio of the rod, $A/V = \frac{4}{d}$
- T_{surr} is the surrounding temperature, $T_{surr} = 300\, K$
- h is an emprical heat transfer coefficient, $h = 40.0 \frac{W}{m^2 \cdot K}$

Answer the following questions and perform the following tasks.

(a) Is this ODE problem linear or nonlinear?
(b) Is this ODE problem an initial value problem or a boundary value problem?
(c) Convert this second order ODE into a system of two first order ODEs.
(d) Find the initial temperature gradient at z = 0.
(e) Sketch the temperature profile.
(f) Verify that your discretization resolution was sufficient.

Problem 2. (8 points)
The Hall-Petch equation states that the yield strength, σ_y, varies with grain size, *d*, according to

$$\sigma_y = \sigma_o + k_y \frac{1}{\sqrt{d}} \tag{1}$$

where σ_o and k_y are material-specific parameters.
[Materials Science and Engineering: An Introduction, 5th Edition, William D. Callister, John Wiley & Sons, Inc., New York, 2000, p. 167.]

For the σ_y vs *d* data given in the file, http://utkstair.org/clausius/docs/mse301/data/xm4p02_f17.txt, perform the following tasks. The grain size is given in mm and the yield strength in MPa.

(a) Identify all variables, $y = mx + b$, when equation (1) is linearized.
(b) Report the best value of σ_o and k_y.
(c) Report the standard deviations of σ_o and k_y.
(d) Report the measure of fit.

Problem 3. (12 points)
The addition of impurities into a metal to improve mechanical properties is known as solid solution strengthening. [Materials Science and Engineering: An Introduction, 5th Edition, William D. Callister, John Wiley & Sons, Inc., New York, 2000, p. 168.] Consider the following data for aluminum and indium solid solutions in magnesium [The Elastic Properties of Magnesium Solid Solutions, D. Hardie, *Acta Metallurgica* Vol. 19, 1971, pp. 719-723]

Aluminum			**Indium**		
sample #	ΔE/atomic %	(ΔE/atomic %)^2	sample #	ΔE/atomic %	(ΔE/atomic %)^2
1	-20.00	400.00	1	-34.37	1181.64
2	-3.33	11.11	2	-23.64	558.68
3	-10.00	100.00	3	-26.39	696.37
4	-10.00	100.00			
sum	-43.33333333	611.1111111		-84.40025253	2436.691768

ΔE = change in Young's modulus relative to pure Mg (kgf/mm^2)

(a) Compute the sample mean of the change in Young's Modulus/atomic % for both metals.
(b) Compute the sample variance of the change in Young's Modulus/atomic % for both metals.
(c) What PDF is appropriate for determining a confidence interval on the difference of means?
(d) Find the lower limit on a 98% confidence interval on the difference of means.
(e) Find the upper limit on a 98% confidence interval on the difference of means.
(f) Translate your result from (d) and (e) into a statement a non-statistician can understand.

Problem 4. (8 points)
Consider the van der Waal's equation of state,

$$p = \frac{RT}{V-b} - \frac{a}{V^2}$$

Consider argon at T=120 K and p = 200,000 Pa. The van der Waals constants for argon are a=0.1381 m^6/mol^2 and b=3.184×10^{-5} m^3/mol. The gas constant is R=8.314 J/mol/K.

(a) Is this algebraic equation linear or nonlinear in the molar volume?
(b) What is the appropriate technique to solve this equation?
(c) Find the liquid molar volume.
(d) Find the vapor molar volume.