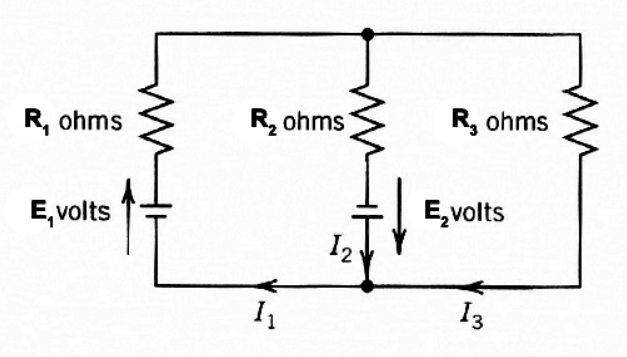
Exam III Solutions
Administered: Wednesday, November 4, 2016
22 points

For each problem part: 0 points if not attempted or no work shown,
1 point for partial credit, if work is shown,
2 points for correct numerical value of solution

Problem 1. (12 points)
Consider the following circuit.



Circuit analysis is performed by using Kirchoff's Current & Voltage laws. Kirchoff's current law states that the sum of the currents entering and leaving any node must sum to zero. So for the bottom node, Kirchoff's current law provides the following equation.

$$-i_1 + i_2 + i_3 = 0$$

where *i* denotes current. Kirchoff's voltage law states that in a loop the sum of the voltages is zero. The application of Kirchoff's voltage law to the right and left loops respectively yields

$$R_1 i_1 + R_2 i_2 = E_1 + E_2$$

$$-R_2 i_2 + R_3 i_3 = -E_2$$

where *R* denotes a resistance and *E* denotes an applied voltage. Consider an example where the following values are assigned: $R_1 = 20,\ R_2 = 10,\ R_3 = 30,\ E_1 = 40,\ E_2 = 50$

(a) Write this set of equations in matrix notation, $\underline{\underline{A}}\,\underline{x} = \underline{b}$. Identify all three quantities, $\underline{\underline{A}}$, $\underline{x}$ and $\underline{b}$.

(b) Calculate the determinant of $\underline{\underline{A}}$.

(c) Calculate the rank of $\underline{\underline{A}}$.

(d) Calculate the rank of $\underline{\underline{A}}|\underline{b}$

(e) How many solutions are there to this problem?

(f) Calculate the currents.

Solution:

(a) Write this set of equations in matrix notation, $\underline{\underline{A}}\,\underline{x} = \underline{b}$. Identify all three quantities, $\underline{\underline{A}}$, $\underline{x}$ and $\underline{b}$.

The three independent equations can be written in matrix form as:

$$\begin{bmatrix} -1 & 1 & 1 \\ R_1 & R_2 & 0 \\ 0 & -R_2 & R_3 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 0 \\ E_1 + E_2 \\ -E_2 \end{bmatrix}$$

To solve the remainder of this problem we can write a short script in a file called xm3p01_f16.m.

```
clear all;
n = 3;
A = zeros(n,n);
x = zeros(n,1);
b = zeros(n,1);
%
R1 = 20;
R2 = 10;
R3 = 30;
E1 = 40;
E2 = 50;
%
A = [-1 1 1
    R1 R2 0
    0  -R2 R3]

b = [0; E1+E2; -E2]

detA = det(A)
rankA = rank(A)
rankAb =rank([A,b])
invA = inv(A);
x = invA*b
```

This script generated the following results.

```
>> xm3p01_f16

A = [   -1    1    1
   20   10    0
    0  -10   30]

b =    [0
   90
  -50]

detA =  -1.1000e+03

rankA =     3
```

rankAb = 3

x = [2.8182
3.3636
-0.5455]

(b) Calculate the determinant of $\underline{\underline{A}}$.

detA = -1.1000e+03

(c) Calculate the rank of $\underline{\underline{A}}$.

rankA = 3

(d) Calculate the rank of $\underline{\underline{A}}|\underline{b}$

rankAb = 3

(e) How many solutions are there to this problem?

This is one solution to this problem.

(f) Calculate the currents.

The currents are given by

x = [2.8182
3.3636
-0.5455]

$i_1 = 2.8182, i_2 = 3.3636, i_3 = -0.5455$

Problem 2. (10 points)
Consider the same circuit as was shown in problem one. However, this time we do not know the value of value of R_2. Instead, we have measured $i_3 = -1.0$.

(a) Is this system of equation linear or nonlinear?
(b) What numerical technique is appropriate for solving this problem?
(c) Solve for the values of i_1, i_2 and R_2.. (6 points)

Kirchoff's voltage and current laws can be written as

$$f_1(i_1, i_2, R_2) = -i_1 + i_2 + i_3 = 0$$

$$f_2(i_1, i_2, R_2) = R_1 i_1 + R_2 i_2 - E_1 - E_2 = 0$$

$$f_3(i_1, i_2, R_2) = -R_2 i_2 + R_3 i_3 + E_2 = 0$$

(a) Is this system of equation linear or nonlinear?

This system of equations is nonlinear because of terms like $R_2 i_2$.

(b) What numerical technique is appropriate for solving this problem?

Multivariate Newton-Raphson Method.

(c) Solve for the values of i_1, i_2 and R_2..

To solve the remainder of this problem we can write a short script in a file called xm3p02_f16.m. We will take our initial guesses from problem 1.

```
clear all;
i1 = 2.8182;
i2 = 3.3636;
R2 = 10.0;
x0 =    [i1; i2; R2];
tol = 1.0e-6;
iprint = 1;
[x,err,f] = nrndn(x0,tol,iprint)
```

In the input subroutine for nrndn.m, I put the following code

```
function f = funkeval(x)
%
% these two lines force a column vector of length n
%
n = max(size(x));
f = zeros(n,1);
%
% enter the functions here
%
i1 = x(1);
i2 = x(2);
```

```
R2 = x(3);
%
R1 = 20;
R3 = 30;
E1 = 40;
E2 = 50;
i3 = -1.0;
%
f(1) = -i1 + i2 + i3;
f(2) = R1*i1 + R2*i2 - E1 - E2;
f(3) = -R2*i2 + R3*i3 + E2;
```

To execute the script, at the Matlab command prompt, I typed

```
>> xm3p2_f16
```

which yielded the following result:

```
>> xm3p02_f16
iter =    1, err =  4.36e+00 f =  7.88e+00
 iter =    2, err =  1.08e+00 f =  6.90e+00
 iter =    3, err =  4.19e-14 f =  2.67e-13

x =
    3.5000
    4.5000
    4.4444

err =   4.1935e-14

f =    2.6687e-13
```

The error indicates that the value converged. So the missing currents and resistance are

$$\underline{x} = \begin{bmatrix} i_1 \\ i_2 \\ R_2 \end{bmatrix} = \begin{bmatrix} 3.5 \\ 4.5 \\ 4.4444 \end{bmatrix}$$