

Exam I
Administered: Wednesday, September 14, 2016
32 points

For each problem part: 0 points if not attempted or no work shown,
 1 point for partial credit, if work is shown,
 2 points for correct numerical value of solution

Problem 1. (12 points)

Materials scientists are evaluating the conductivity (S/cm) of a proton exchange membrane composed of a blend of two polymers, Nafion & poly-ethylene glycol (PEG) as a function of PEG content (wt%) at a fixed temperature and relative humidity.

sample	PEG content (wt %)	conductivity (S/cm)	PEG ² (wt % ²)	conductivity ² (S ² /cm ²)	PEG*conductivity (wt %-S/cm)
1	0.00	0.10	0.00	0.01	0.00
2	0.00	0.09	0.00	0.01	0.00
3	0.25	7.81	0.06	61.00	1.95
4	0.25	6.93	0.06	48.02	1.73
5	0.50	10.00	0.25	100.00	5.00
6	0.50	9.82	0.25	96.43	4.91
7	0.75	0.03	0.56	0.00	0.02
8	0.75	0.04	0.56	0.00	0.03
9	1.00	0.00	1.00	0.00	0.00
10	1.00	0.01	1.00	0.00	0.01
sum	5.0000	34.8300	3.7500	305.4741	13.6575

Based on this data, answer the following questions.

- (a) Find the variance of the conductivity.
- (b) Find the standard deviation of the conductivity.
- (c) Find the covariance of the PEG content and conductivity.
- (d) Find the correlation coefficient of the PEG content and conductivity.
- (e) Are the PEG content and conductivity independent random variables?
- (f) What is the physical relationship between the PEG content and conductivity?

(over)

Problem 2. (10 points)

A study of blended membranes discovered that the PEG and Nafion were not uniformly mixed for a given processing condition. In fact, the PEG content within the membrane is given by the following distribution

$$f(w_{PEG}) = \begin{cases} k(w_{PEG} - w_{PEG}^2) & \text{for } 0.0 \leq w_{PEG} \leq 1.0 \\ 0 & \text{otherwise} \end{cases}$$

The conductivity, c , can be approximately related to the local PEG content via the function

$$c(w_{PEG}) = 34.0(w_{PEG} - w_{PEG}^2) \text{ S/cm} \quad \text{for } 0.0 \leq w_{PEG} \leq 1.0$$

- Is the PDF continuous or discrete?
- Find the value of k that normalizes this PDF.
- Find the probability a portion of the membrane contains a PEG content less than 0.3.
- Find the probability a portion of the membrane contains a PEG content greater than 0.3.
- Find the average conductivity.

Problem 3. (10 points)

A study is performed involving fuel cell containing a proton exchange membrane composed of a homopolymer (H) or a blend (B) of two polymers. Each fuel cell is evaluated for a one hundred cold start cycles and is categorized as either (P) or (F).

During this study the following information was collected.

- # of fuel cells with a homopolymer membrane that passed the test = 10
- # of fuel cells with a homopolymer membrane that failed the test = 3
- # of fuel cells with a blend membrane that passed the test = 6
- # of fuel cells with a blend membrane that failed the test = 1

Using this information, answer the following questions.

- Draw a Venn Diagram of the sample space for this experiment.
- What is the probability that a fuel cell in this study contained a homopolymer membrane?
- What is the probability that a fuel cell passed the test given that it contained a homopolymer membrane?
- What is the probability that a fuel cell passed the test given that it contained a blend membrane?
- What is the probability that the membrane contained the blend given that the test failed?