

Exam I

Administered: Wednesday, September 16, 2015

32 points

For each problem part: 0 points if not attempted or no work shown,
 1 point for partial credit, if work is shown,
 2 points for correct numerical value of solution

Problem 1. (12 points)

Materials Scientists students are polishing 8 ceramic samples with an abrasive with a 320 grit. The following table provides the length of time the sample was polished (s) and the RMS surface roughness (μm).

sample	time (s)	surface roughness (μm)	time ² (s ²)	surface roughness ² (μm^2)	time*surface roughness (s· μm)
1	60	1.7	3600	2.89	102
2	60	1.8	3600	3.24	108
3	120	0.8	14400	0.64	96
4	120	0.68	14400	0.4624	81.6
5	180	0.46	32400	0.2116	82.8
6	240	0.34	57600	0.1156	81.6
7	300	0.21	90000	0.0441	63
8	600	0.24	360000	0.0576	144
sum	1680	6.23	576000	7.6613	759

Based on this data, answer the following questions.

- Find the variance of the surface roughness.
- Find the standard deviation of the surface roughness.
- Find the covariance of polishing time and surface roughness.
- Find the correlation coefficient of polishing time and surface roughness.
- Are the polishing time and surface roughness independent random variables?
- What is the physical relationship between polishing time and surface roughness based on your answer to (d)?

Problem 2. (10 points)

A study of students in a polishing laboratory discover that, regardless of the recommended polishing time, the amount of time students polish a sample, t , is historically given by the following probability distribution function.

$$f(t) = \begin{cases} c/t & \text{for } 60 \leq t \leq 600 \\ 0 & \text{otherwise} \end{cases}$$

The surface roughness, s , can be approximately related to the polishing time via the function

$$s(t) = 120/t \text{ } \mu\text{m} \quad \text{for } 60 \leq t \leq 600$$

- Is the PDF continuous or discrete?
- Find the value of c that normalizes this PDF.
- Find the probability a student polishes a sample less than five minutes.
- Find the probability a student polishes a sample more than five minutes.
- Find the average surface roughness.

Problem 3. (10 points)

A manufacturer of abrasives intended for polishing is evaluating two competing products, labeled A and B. Product A comes with instructions that say the abrasive should be used for at least five minutes, while product B is to be used for at least 10 minutes. They issued a survey asking users of both products whether they polished for the recommended time. The following information was collected.

of respondents who used abrasive A for at least the recommended polishing time = 50
of respondents who used abrasive A for less than the recommended polishing time = 20
of respondents who used abrasive B for at least the recommended polishing time = 10
of respondents who used abrasive B for less than the recommended polishing time = 20

Using this information, answer the following questions.

- (a) Draw a Venn Diagram of the sample space for this experiment.
- (b) What is the probability that a person bought abrasive A?
- (c) What is the probability that a person sufficiently polished a sample given that they bought abrasive A?
- (d) What is the probability that a person sufficiently polished a sample given that they bought abrasive B?
- (e) What is the probability that a person bought abrasive B given that they sufficiently polished a sample?