Exam II
Administered: Tuesday, October 15, 2013
24 points

For each problem part: 0 points if not attempted or no work shown,
1 point for partial credit, if work is shown,
2 points for correct numerical value of solution

Problem 1. (8 points)

A particular device is powered by three batteries. Each battery has a mean life time of 10 months. The device only operates if all three batteries continue to function.

(a) What PDF would describe the probability that an individual battery is operating after 6 months?
(b) What is the probability that an individual battery is operating after 6 months?
(c) What PDF would describe the probability that all 3 batteries are functioning after 6 months?
(d) What is the probability that the device still operates after 6 months?

Solution:

(a) What PDF would describe the probability that an individual battery is operating after 6 months?

The exponential PDF would describe the life-time of a single battery.

(b) What is the probability that an individual battery is operating after 6 months?

$$P(t>6)=\int_6^\infty f_e(t;\beta)dt=\int_6^\infty \frac{1}{\beta}e^{-\frac{t}{\beta}}dt=-e^{-\frac{t}{\beta}}\Big|_6^\infty=-e^{-\frac{\infty}{\beta}}--e^{-\frac{6}{\beta}}=0+e^{-\frac{6}{\beta}}$$

$$P(t>6)=e^{-\frac{6}{10}}=0.5488$$

(c) What PDF would describe the probability that all 3 batteries are functioning after 6 months?

This calls for the binomial probability because the batteries are independent and all have the same probability of operating after 6 months have passed.

(d) What is the probability that the device still operates after 6 months?

x = random variable = number of batteries operating after 6 months = 3
n = total number of batteries = 3
p = probability that an individual battery is operating after 6 months = answer to part (b)

$$P(X=x)=b(x;n,p)=\binom{n}{x}p^x q^{n-x}$$

$$P(x=3)=b(3;3,0.5488)=0.1653$$

Problem 2. (6 points)

We run a warranty company that provides replacement parts for digital cameras. If our research team tells us that on average digital cameras have a lifetime of 4 years with a standard deviation of 1.5 years, then answer the following questions.

(a) If we provide a warranty for all cameras lasting less than 1 years, what fraction of the cameras can we expect to replace?

(b) If we only want to replace 1% of the cameras, how long should our warranty last?

(c) What PDF did you use to solve (a) & (b)?

Solution:

(a) If we provide a warranty for all cameras lasting less than 1 years, what fraction of the cameras can we expect to replace?

$$P(x<1) = P(z < \frac{x-\mu}{\sigma}) = P(z < \frac{1-4}{1.5}) = P(z < -2)$$

```
>> p = cdf('normal',-2,0,1)

p =   0.022750131948179
```

$$P(x<1) = P(z < \frac{x-\mu}{\sigma}) = P(z < \frac{1-4}{1.5}) = P(z < -2) = 0.0228$$

We can expect 2.28% of the cameras to fail before one year.

(b) If we only want to replace 1% of the cameras, how long should our warranty last?

$$P(z < z_{lo}) = 0.01$$

```
>> z = icdf('normal',0.01,0,1)

z =  -2.326347874040841
```

$$z_{lo} = \frac{x_{lo} - \mu}{\sigma}$$

$$x_{lo} = \mu + \sigma z_{lo} = 4 + 1.5(-2.326) = 0.5104 \text{ years}$$

Our warranty should last 0.5104 years.

(c) What PDF did you use to solve (a) & (b)?

Normal distribution. The variable is continuous and we have only been provided the mean and the standard deviation.

Problem 3. (10 points)

We are in charge of designing a secondary containment area for an area surrounding a series of large tanks containing concentrated sulfuric acid. The purpose of the containment area is to hold the sulfuric acid for a relatively short time in the event that one of the tanks springs a major leak. This containment area is to be concrete lined with a corrosion-resistant film. We are examining 2 types of films.

Film 1 is polycarbonate based. Studies of 12 experiments indicate that the average contact time before the film fails is 24 hours with a sample standard deviation of 2 hours.

Film 2 is a polymer/silica gel composite material. Studies of 16 experiments indicate that the average contact time before the film fails is 30 hours with a sample standard deviation of 4 hours.

A square foot of Film 2 is twice as expensive as a square foot of Film 1.

Based on this information, answer the following questions.

(a) What PDF is appropriate for determining a confidence interval on the difference of means?
(b) Find the lower limit on a 96% confidence interval on the difference of means.
(c) Find the upper limit on a 96% confidence interval on the difference of means.
(d) If our boss says that in order to justify the higher cost of Film 2, then we need to be 96% confident that Film 2 lasts at least 3 hours longer than Film 1, which film do we recommend? Why?
(e) How confident are we that Film 2 lasts 3 hours longer than Film 1?

Solution:

(a) What PDF is appropriate for determining a confidence interval on the difference of means?

We must use the t-distribution to determine the confidence interval on the difference of means when the population variances are unknown.

(b) Find the lower limit on a 96% confidence interval on the difference of means.
(c) Find the upper limit on a 96% confidence interval on the difference of means.

$$\bar{x}_1 = 24 \qquad \bar{x}_2 = 30 \qquad n_1 = 12 \qquad n_2 = 16 \qquad s_1 = 2 \qquad s_2 = 4$$

$$\alpha = \frac{1 - C.I.}{2} = 0.02$$

$$v = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\left[\left(\frac{s_1^2}{n_1}\right)^2 \Big/ (n_1 - 1)\right] + \left[\left(\frac{s_2^2}{n_2}\right)^2 \Big/ (n_2 - 1)\right]} \text{ if } \sigma_1 \neq \sigma_2$$

$$v = 23.158 \approx 23$$

Use the icdf command in MatLab to calculate the probability.

```
>> p = icdf('t',0.02,23)

p =  -2.176958111315391
```

Use symmetry to compute the other t value.

$$t_{1-\alpha} = -t_\alpha$$

confidence interval:

$$P\left[\left(\overline{X}_1-\overline{X}_2\right)+t_{\alpha}\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}<\left(\mu_1-\mu_2\right)<\left(\overline{X}_1-\overline{X}_2\right)+t_{1-\alpha}\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}\right]=1-2\alpha$$

$$P\left[\left(24-30\right)+\text{-}2.177\sqrt{\frac{2^2}{12}+\frac{4^2}{16}}<\left(\mu_1-\mu_2\right)<\left(24-30\right)+2.177\sqrt{\frac{2^2}{12}+\frac{4^2}{16}}\right]=0.96$$

$$P\left[\text{-}8.514<\left(\mu_1-\mu_2\right)<\text{-}3.486\right]=0.96$$

(d) If our boss says that in order to justify the higher cost of Film 2, then we need to be 96% confident that Film 2 lasts at least 3 hours longer than Film 1, which film do we recommend? Why?

We recommend Film 2 because our 96% confident interval is entirely less than -3.

(e) How confident are we that Film 2 lasts at least 3 hours longer than Film 1?

This is the reverse problem. This is the given-t, find p problem.

$$T_{lo}=\frac{\left(\overline{X}_1-\overline{X}_2\right)-\left(\mu_1-\mu_2\right)}{\sqrt{\left(\frac{s_1^2}{n_1}\right)+\left(\frac{s_2^2}{n_2}\right)}}=\frac{\left(-6\right)-\left(-3\right)}{1.1547}=\text{-}2.5981$$

We want the probability that

$$P(-T_{lo}<t;v=23)$$

Since we have a code that delivers the cumulative probability, we rewrite this as

$$P(-T_{lo}<t;v=23)=1-P(t<-T_{lo};v=23)$$

We use cdf function in MatLab.

$$P(-T_{lo}<t;v=23)=1-P(t<-T_{lo};v=23)=1-0.00804=0.99196$$

We are 99.2% confident that Film 2 lasts at least 3 hours longer than Film 1.