Final Exam
Administered: Thursday, December 12, 2013
24 points

Problem 1. (8 points)

Consider the isomerization reaction:

$$A \rightarrow B$$

The reaction rate is given by

$$rate = C_A k_o e^{-\frac{E_a}{RT}} \qquad \text{[moles/m}^3\text{/sec]}$$

where

concentration of A: C_A [moles/m^3]
prefactor: k_o [1/sec]
activation energy for reaction: E_a [Joules/mole]
constant: $R = 8.314$ [Joules/mole/K]
temperature: T [K]

Determine the rate constants, k_o and E_a, from experimental data. The reaction is measured at a constant concentration of A, C_A = 2000 mol/m^3, over a range of temperatures. The rate is recorded. The rate as a function of temperature is given in tabular form in the file "xm4p01_f13.txt" on the exam portion of the course website.

Problem 2. (8 points)

The complementary error function is defined as

$$erfc(x) = 1 - \frac{2}{\sqrt{\pi}} \int_0^x \exp\left(-t^2\right) dt$$

(a) Evaluate the complementary error function for x = 2 using the intrinsic erfc function. You likely will need to use the `format long` statement in MatLab to get enough digits to display.
(b) How many intervals do you need in the second-order Simpson's method to obtain this result to four significant digits?

Problem 3. (8 points)

The one-dimensional heat equation at steady state can describe heat transfer in a material with both heat conduction and radiative heat loss.

$$0 = \frac{k}{\rho C_p}\frac{d^2T}{dz^2} - \frac{\varepsilon\sigma S}{\rho C_p}\left(T^4 - T_s^{\,4}\right)$$

where the following variables [with units] are given as

temperature in the material T [K]
surrounding temperature $T_s = 300$ [K]
axial position along material z [m]
thermal conductivity $k = 401$ [J/K/m/s] (for Cu)
mass density $\rho =$ 8960 [kg/m^3] (for Cu)
heat capacity $C_p = 384.6$ [J/kg/K] (for Cu)
Stefan–Boltzmann constant $\sigma = 6.6404x10^{-8}$ [J/s/m^2/K^4]
gray body permittivity $\varepsilon = 0.05$
surface area to volume ratio $S = 200$ [1/m] (for a cylindrical rod of diameter 0.01 m)

If the temperature of the material at z=0 is $T(z=0) = 1000$ K and the flux at z=0 is $\left.\frac{dT}{dz}\right|_{z=0} = -2000$ K/m, find the temperature in the material at z=0.1 m.