Exam III
Administered: Thursday, November 14, 2013
28 points

For each problem part: 0 points if not attempted or no work shown,
1 point for partial credit, if work is shown,
2 points for correct numerical value of solution

**Problem 1. (14 points)**
Consider the matrix and vector:

$$\underline{\underline{A}} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 1 & 0 \\ 1 & 0 & -2 & 1 \\ 0 & 1 & 1 & -2 \end{bmatrix} \qquad \underline{b} = \begin{bmatrix} 10 \\ 8 \\ -1 \\ -3 \end{bmatrix}$$

Report the following information.

(a) the determinant of $\underline{\underline{A}}$

(b) the rank of $\underline{\underline{A}}$

(c) the rank of the augmented $\underline{\underline{Ab}}$ matrix

(d) the number of solutions to $\underline{\underline{A}}\underline{x} = \underline{b}$

(e) the inverse of $\underline{\underline{A}}$ if it exists

(f) a solution to $\underline{\underline{A}}\underline{x} = \underline{b}$, if it exists

(g) the eigenvalues of $\underline{\underline{A}}$

**Problem 2. (6 points)**
Use the Newton-Raphson method with numerical approximations to the derivative to find the molar volume(s) of ammonia from the the van der Waals equation of state at $T = 500$ K and p = 1013250 bar.
The van der Waals equation of state is given by

$$P = \frac{RT}{V-b} - \frac{a}{V^2}$$

where, $R = 8.314$ J/mol/K. The van der Waals constants for ammonia are $a = 0.4253$ m$^6$/mol$^2$, $b = 3.737x10^{-5}$ m$^3$/mol. The critical temperature of ammonia is $T_c = 430.6$ K.

**Problem 3. (8 points)**
Use the multivariate Newton-Raphson method with numerical approximations to the derivative to find the solution near [1,1,1] to this set of non-linear algebraic equations

$$f_1 = x_1 + 2x_2 + 3x_3 - 7$$
$$f_2 = x_1^3 - 4x_2^3 + 10x_3^3 - 10$$
$$f_3 = x_2 - \exp(x_3)$$