Exam II
Administered: Wednesday, October 10, 2001
30 points

For each problem part: 0 points if not attempted or no work shown,
1 point for partial credit, if work is shown,
2 points for correct numerical value of solution

Problem 1. (10 points)

We are in charge of designing a secondary containment area for an area surrounding a series of large tanks containing concentrated nitric acid. The purpose of the containment area is to hold the nitric acid for a relatively short time in the event that one of the tanks springs a major leak. This containment area is to be concrete lined with a corrosion-resistant films. We are examining 2 types of films.

Film 1 is polycarbonate based. Studies of 12 experiments indicate that the average contact time before the film fails is 11 hours with a sample standard deviation of 1.5 hours.

Film 2 is a polymer/silica gel composite material. Studies of 16 experiments indicate that the average contact time before the film fails is 15 hours with a sample standard deviation of 3 hours.

A square foot of Film 2 is twice as expensive as a square foot of Film 1.

Based on this information, answer the following questions.

(a) What PDF is appropriate for determining a confidence interval on the difference of means?
(b) Find the lower limit on a 96% confidence interval on the difference of means.
(c) Find the upper limit on a 96% confidence interval on the difference of means.
(d) If our boss says that in order to justify the higher cost of Film 2, then we need to be 96% confident that Film 2 lasts 2 hours longer than Film 1, which film do we recommend? Why?
(e) How confident are we that Film2 lasts 3 hours longer than Film 1?

Solution:

(a) What PDF is appropriate for determining a confidence interval on the difference of means?

We must use the t-distribution to determine the confidence interval on the difference of means when the population variances are unknown.

(b) Find the lower limit on a 96% confidence interval on the difference of means.
(c) Find the upper limit on a 96% confidence interval on the difference of means.

$$\bar{x}_1 = 11 \qquad \bar{x}_2 = 15 \qquad n_1 = 12 \qquad n_2 = 16 \qquad s_1 = 1.5 \qquad s_2 = 3$$

$$\alpha = \frac{1 - \text{C.I.}}{2} = 0.02$$

$$v = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\left[\left(\frac{s_1^2}{n_1}\right)^2 \Big/ (n_1 - 1)\right] + \left[\left(\frac{s_2^2}{n_2}\right)^2 \Big/ (n_2 - 1)\right]} \text{ if } \sigma_1 \neq \sigma_2$$

$$v = 23.158 \approx 23$$

$$t_\alpha(v) = t_{0.02}(23) = 2.177 \text{ from Table A.4 of WMM}$$

$$t_{1-\alpha} = -t_\alpha$$

confidence interval:

$$P\left[\left(\overline{X}_1 - \overline{X}_2\right) + t_{1-\alpha}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} < \left(\mu_1 - \mu_2\right) < \left(\overline{X}_1 - \overline{X}_2\right) + t_{\alpha}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}\right] = 1 - 2\alpha$$

$$P\left[-5.8853 < \left(\mu_1 - \mu_2\right) < -2.1147\right] = 0.96$$

(d) If our boss says that in order to justify the higher cost of Film 2, then we need to be 96% confident that Film 2 lasts 2 hours longer than Film 1, which film do we recommend? Why?

We recommend Film 2 because we are 96% confident that Film 2 lasts at least 2.11 hours longer than Film 1.

(e) How confident are we that Film2 lasts at least 3 hours longer than Film 1?

This is the reverse problem. This is the given-t, find p problem.

$$T_{lo} = \frac{\left(\overline{X}_1 - \overline{X}_2\right) - \left(\mu_1 - \mu_2\right)}{\sqrt{\left(\frac{s_1^2}{n_1}\right) + \left(\frac{s_2^2}{n_2}\right)}} = \frac{(-4) - (-3)}{0.8660} = -1.1547$$

The table does not contain negative values for T. However, due to the symmetry of the T-distribution, we can see that

$$P(-T_{lo} < t; v = 23) = 0.13$$

By symmetry,

$$P(T_{lo} > t; v = 23) = 0.13$$

So, the probability that we want is

$$P(T_{lo} < t; v = 23) = 1 - P(T_{lo} > t; v = 23) = 1 - 0.13 = 0.87$$

We are 87% confident that Film 2 lasts at least 3 hours longer than Film 1.

Problem 2. (8 points)

A particular process is continues to operate so long as at least one of four circuit breakers is in operation. The circuit breakers control the same operation in parallel and are independent of each other. Individually, the circuit breakers have a mean duration of operation of 150 hours before they need to be reset.

(a) What PDF would describe the probability that an individual circuit breaker is operating after one week?

(b) What is the probability that an individual circuit breaker is operating after one week?

(c) What PDF would describe the probability that 3 of 4 circuit breakers are operating after one week?

(d) What is the probability that 3 of 4 circuit breakers are operating after one week?

Solution:

(a) What PDF would describe the probability that an individual circuit breaker is operating after one week?

The exponential PDF would describe the life-time of a single circuit breaker.

(b) What is the probability that an individual circuit breaker is operating after one week?

1 week = 168 hours, β = mean life = 150 hours

$$P(t > 168) = \int_{168}^{\infty} f_e(t;\beta)dt = \int_{168}^{\infty} \frac{1}{\beta} e^{-\frac{t}{\beta}} dt = -e^{-\frac{t}{\beta}}\Bigg|_{168}^{\infty} = -e^{-\frac{\infty}{\beta}} - -e^{-\frac{168}{\beta}} = 0 + e^{-\frac{168}{\beta}}$$

$$P(t > 168) = e^{-\frac{168}{150}} = 0.3263$$

(c) What PDF would describe the probability that 3 of 4 circuit breakers are operating after one week?

This calls for the binomial probability because the circuit breakers are independent and all have the same probability of operating after a week has passed.

(d) What is the probability that 3 of 4 circuit breakers are operating after one week?

x = random variable = number of breakers operating after one week = 3
n = total number of breakers = 4
p = probability that an individual breaker is operating after one week = answer to part (b)

$$P(X = x) = b(x;n,p) = \binom{n}{x} p^x q^{n-x}$$

$$P(x = 3) = b(3;4,0.3263) = 0.0936$$

Problem 3. (8 points)

We run a warranty company that provides replacement parts for digital cameras. If our research team tells us that on average digital cameras have a lifetime of 5 years with a standard deviation of 2 years, then answer the following questions.

(a) If we provide a warranty for all cameras lasting less than 3 years, what fraction of the cameras can we expect to replace?

(b) If we only want to replace 1% of the cameras, how long should our warranty last?

(c) What PDF did you use to solve (a) & (b)?

(d) If we want our warranty program simply to break even, and the average cost of a digital camera is \$300. How much should we charge for the 3-year (part b) warranty protection?

Solution:

(a) If we provide a warranty for all cameras lasting less than 3 years, what fraction of the cameras can we expect to replace?

$$P(x < 3) = P(z < \frac{x-\mu}{\sigma}) = P(z < \frac{3-5}{2}) = P(z < -1) = 0.1587$$

From table A.3.
We can expect 15.87 % of the cameras to fail before one year.

(b) If we only want to replace 1% of the cameras, how long should our warranty last?

$$P(z < z_{lo}) = 0.01$$

$z_{lo} = -2.327$ from table A.3

$$z_{lo} = \frac{x_{lo} - \mu}{\sigma}$$
$$x_{lo} = \mu + \sigma z_{lo} = 5 + 2(-2.327) = 0.346 \text{ years}$$

Our warranty should last 0.346 years or 4.15 months or 126 days.

(c) What PDF did you use to solve (a) & (b)?

Normal distribution. The variable is continuous and we have only been provided the mean and the standard deviation.

(d) If we want our warranty program simply to break even, and the average cost of a digital camera is \$300. How much should we charge for the warranty protection?

Since a camera costs \$300 and 15.87% of the cameras fail, we should charge \$47.61 in order to break even.

Problem 4. (4 points)

(a) What is the probability of getting five or fewer heads when flipping a coin ten times?
(b) Why isn't the probability of getting five or fewer heads when flipping a coin ten times equal to ½?

Solution:

(a) What is the probability of getting five or fewer heads when flipping a coin ten times?

This calls for binomial distribution.

$$P(x \le 5) = \sum_{x=0}^{5} b(x;10,\frac{1}{2}) = 0.6230$$

(b) Why isn't the probability of getting five or fewer heads when flipping a coin ten times equal to ½?

There are eleven possible values of x, from 0, 1, 2, … 9, 10. Six of these eleven values are included in "five or fewer heads". Therefore, we can hardly expect that taking six of eleven outcomes will give us a probability of ½. While there is symmetry in the binomial distribution, the middle point, x=5, does not have a symmetric partner. This can most easily be seen from a histogram of the PDF.