Homework Assignment Number Five
Assigned: Tuesday, October 26, 1999
Due: Tuesday, November 9, 1999 BEGINNING OF CLASS.

Consider the following one-dimensional linear parabolic P.D.E., commonly known as the heat equation:

$$\frac{\partial T}{\partial t} = \alpha\left(\frac{\partial^2 T}{\partial x^2}\right) + \frac{f(t,x,y,z)}{\rho \hat{C}_p}$$

where , α , the thermal diffusivity is defined as

$$\alpha = \frac{k}{\rho \hat{C}_p}$$

(1) Consider an aluminum cylindrical rod 1.0 meter long connecting two heat reservoirs. One of the reservoirs is maintained at T=300K, the other reservoir at T=400 K. Initially, the cylinder is at 300 K. There is no heat loss from the rod. Consider the system to be one-dimensional.

(a) Write the IC and BC's.
(b) What does the initial profile look like?
(c) What does the steady state profile look like? Explain.
(d) What is the temperature 0.5 meters into the rod at steady state?
(e) What is the temperature 0.5 meters into the rod after 1000 seconds?
(f) Approximately how long does it take for the midpoint of the rod to get within 1% of the steady state value?
(g) Approximately how long does it take for the midpoint of a lead (Pb) rod to get within 1% of the steady state value? Explain the difference.

(2) Consider an aluminum cylindrical rod 1.0 meter long with one end connected to a heat reservoir at T=400 K. The other end is insulated. The entire rod is also insulated so that there is no heat loss to the surroundings. The initial temperature of the rod is 300 K.

(a) Write the IC and BC's.
(b) What does the steady state profile look like? Explain.
(d) What is the temperature at the free end of the rod at steady state?
(e) What is the temperature 0.5 meters into the rod after 1000 seconds?
(f) Approximately how long does it take for the end of the rod to get within 1% of the steady state value?
(g) Approximately how long does it take for the end of a gold (Au) rod to get within 1% of the steady state value?

(3) Consider an aluminum cylindrical rod 1.0 meter long connecting two heat reservoirs. One of the reservoirs is maintained at T=300K, the other reservoir at T=400 K. Initially, the cylinder is at 300 K. The rod is not insulated so heat is lost from the rod, which has a radius of 5.0 cm. The surrounding temperature is 200 K. Use a heat transfer coefficient of 40.0 $W/m^2/K$. Consider the system to be one-dimensional.

(a) Write the IC and BC's.
(b) What does the steady state profile look like? Explain.
(c) What is the temperature 0.5 meters into the rod at steady state?
(d) What is the temperature 0.5 meters into the rod after 1000 seconds?
(e) Approximately how long does it take for the midpoint of the rod to get within 1% of the steady state value?
(f) Approximately how long does it take for the midpoint of a copper (Cu) rod to get within 1% of the steady state value? Explain the difference.