**Homework Assignment Number Four**
**Assigned: Thursday, October 7, 1999**
**Due: Tuesday, October 19, 1999 BEGINNING OF CLASS.**

**Problem (1)**

Consider the initial value problem:

$$\frac{dy}{dx} + a(x)y = b(x)$$

where we have an initial condition of the form:

$$y(x = x_o) = y_o$$

with the specific values given by:

$$a(x) = 2\,,\ b(x) = x\sin(3x)\,,\ y(x = 0) = 1$$

In homework assignment three, you analytically solved and plotted the solution from x = 0 to 4. Now numerical solve same equation.

(a) Use Euler with a time step of 0.4
(b) Use Euler with a time step of 0.04
(c) Use Runge-Kutta with a time step of 0.4
(d) Use Runge-Kutta with a time step of 0.04
(e) Compare the relative error of the Euler estimate of y(x=4) for both sized steps. Explain.
(f) Compare the relative error of the Runge-Kutta estimate of y(x=4) for both sized steps. Explain.
(g) Compare the relative errors of the Euler and Runge-Kutta estimates of y(x=4) for a time step of 0.04. Explain.
(h) Compare the relative errors of the Runge-Kutta estimates of y(x=2) and y(x=4) for a time step of 0.04. Explain.

**Problem (2)**

In Homework assignment 3, you plotted the solution to the acetylene vibrational/translational problem for the initial conditions:

$$\underline{x}(t = 0) = \begin{bmatrix} -0.1 \\ 0 \\ 0.25 \\ 0.5 \end{bmatrix} \text{ and initial velocities } \dot{\underline{x}}(t = 0) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

for t = 0 to 2 sec, using the following values for the masses and the spring constants.

$m_H = 1.0$, $m_C = 12.0$, $k_{HC} = 1.0$, $k_{CC} = 10.0$

(a) Solve the problem using the Runge-Kutta fourth-order method.
(b) Compare the analytical and numerical solutions.

**Problem (3)**

Consider:

$$\frac{d^2y}{dx^2} = c_1 \frac{dy}{dx} + c_2 y + c_3 \sin(x) + c_4$$

with the boundary conditions

$$y(x=0) = y_o = 1.0$$
$$y(x=10) = y_f = 1.0$$

where $c = [1.0, -2.0, 2.0, 0.0,]$

(a) Convert this single second-order ODE, to a system of two first-order ODEs.
(b) Determine the behavior of $y(x)$ and $y'(x)$ from $0 \le x \le 10$. Show the behavior in a plot. Clearly identify which curve corresponds to which function.
(c) State what value of an initial guess for $y'(x=0)$ led to the final solution.

**Problem (4)**

Find an application from your own experience or a classical problem in your own field of research that results in a ODE boundary value problem that can be solved using the shooting method.

(a) Describe the physical problem from which the equation arises. Describe it in sufficient detail that an engineer from a different discipline could understand it.

(b) Write the ODE(s). Write a complete set of reasonable boundary conditions.

(c) If possible, analytically solve for the solution(s). Otherwise numerically solve the for the solution.

(d) Plot the solution.

(e) Explain the physical significance of the solution(s) and its behavior.