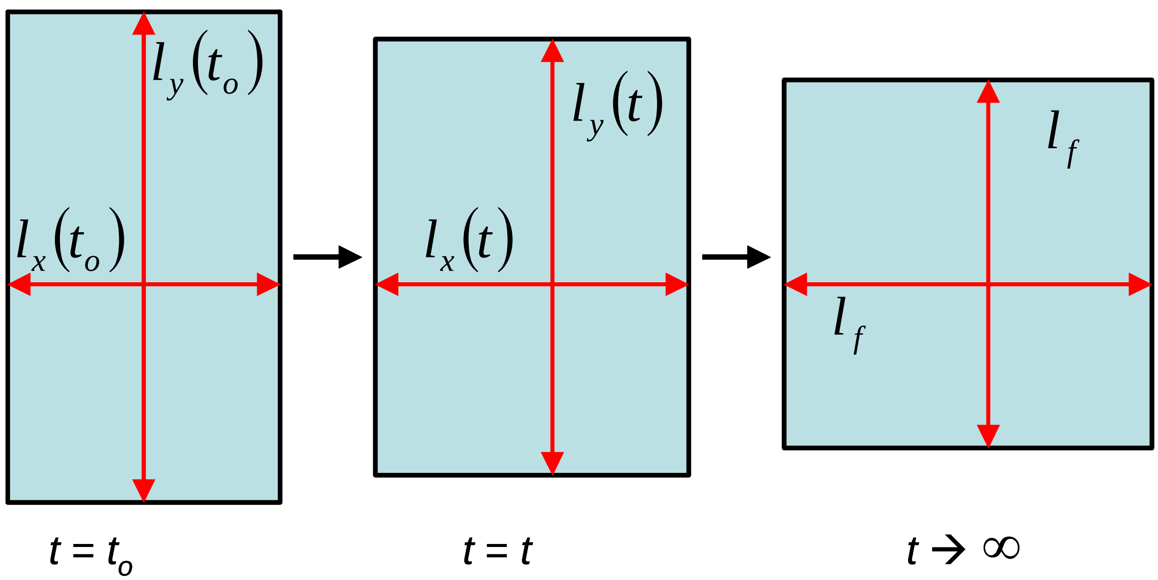
ChE/MSE 505: Final Examination
Administered: Friday, December 7, 2007

Problem (1)

Consider a crystalline material of density, ρ, that has initial dimensions $l_x(t_o)$ and $l_y(t_o)$. (In this problem we will limit ourselves to two dimensions.) This material has a high (unfavorable) surface energy and therefore would like to change its shape (through diffusion or convection, the actual physical mechanism doesn't matter for this problem) to a shape that has the minimum perimeter, namely $l_x(t \to \infty) = l_y(t \to \infty) = l_f$. This process is illustrated in the figure below.



The rate of change of the perimeter, $P(t) = 2l_x(t) + 2l_y(t)$, can be assumed to follow a first order rate law.

$$\frac{dP}{dt} = -\frac{1}{\tau}\left[P(t) - P_f\right]$$

where τ is the characteristic relaxation time associated with the process (assume τ is constant) and P_f is the perimeter at infinite time, $P_f = 4l_f$. This relaxation process occurs under the constraint of conservation of mass. The mass of the system at any time is given by

$$M = \rho A(t) = \rho\left[l_x(t) l_y(t)\right]$$

where $A(t)$ is the area of the material, $A(t) = l_x(t) l_y(t)$. From the conservation of mass, we know that the mass at the initial time and at infinite time are the same

$$M(t_o) = \rho\left[l_x(t_o) l_y(t_o)\right] = M(t \to \infty) = \rho\left[l_f^2\right]$$

Equating these, we can solve for l_f which is a parameter in the rate law.

$$l_f = \sqrt{l_x(t_o) l_y(t_o)}$$

This is the complete information required to describe the evolution of the material shape in time. Based on this information, answer the following questions.

(a) What kind of equations (AE, ODE, PDE or IE) do you have in this system?
(b) What is the independent variable in this system?
(c) What are the dependent variables that must be solved for in this system?
(d) Are the equations linear or nonlinear in the unknowns?
(e) What are the equations in terms of the dependent variables that must be solved for in this system?
(f) Convert the system of equations into a form that you know how to numerically solve.
(g) What conditions (initial conditions or boundary conditions) are required to solve this problem? What are they?
(h) What numerical technique would use you to solve this problem?
(i) Show that $l_x(t \to \infty) = l_y(t \to \infty) = l_f$ is indeed a critical point of the system.
(j) Extra credit: Determine the stability of the critical point and the type of critical point.