Applied Statistics and Numerical Methods for Engineers
ChE 301, Spring 1999
Exam Number Four
Administered: 8:00-10:00 A.M. Thursday May 6, 1999

The exam has 12*10=120 points. Reserve approximately 10 minutes for each problem.

Problem 1. Random Variables

Given the joint PDF

	y				
x	1	2	3	4	5
0	0.01	0.02	0.03	0.04	0.05
0.25	0.02	0.03	0.04	0.05	0.11
0.5	0.03	0.04	0.05	0.06	0.02
0.75	0.04	0.05	0.06	0.02	0.03
1.0	0.05	0.06	0.02	0.03	0.04

Find $P(0 \le x \le 0.5, 2 < y \le 4)$

Problem 2. Expectations

Given the PDF, $f(x) = \frac{x^2}{9} \quad \text{for} \quad 0 \le x \le 3$, find the population variance of x.

Problem 3. Discrete Distributions

In sampling a liquid product from the assembly line with a historical defect rate of 12%, what is the probability of finding that your seventh sample turns out to be your second defect?

Problem 4. Continuous Distributions

A steady temperature of an ink cartridge in a copier is essential for proper performance of the copier. If the temperature deviates more than 2 degrees Celsius from the desired temperature, the copying process fails. Consider a copier with a standard deviation of 0.8 degrees Celsius cartridge temperature. What percent of the time will the copier fail to function properly (excluding all other sources of copier mishaps)?

Problem 5. Sampling and Estimation

Scientists are now saying that the recent outbreak of deformed frogs in Minnesota ponds is due to the increased presence of a certain type of bacteria, (possibly itself due to an increase in agriculture chemical run-off into the pond, which stimulates algae growth and increases the bacteria population). A scientist claims that 0.08 fraction of the total MN frog population has some sort of deformity.

The elementary school kids who made the initial discovery of the deformed frogs, found 12% of collected frogs to have deformities, with a sample standard deviation of 3% based on 16 samples.

Determine whether we can believe the scientist claim within a 95% confidence interval, based on the children's sampling results.

Problem 6. Linear Algebra

Consider an nxn matrix, $\underline{\underline{J}}$, with rank = n-1. Indicate which of any of the following statements are true.

(a) The inverse of $\underline{\underline{J}}$ exists.

(b) At least 2 rows of $\underline{\underline{J}}$ are linearly dependent.

(c) The determinant of $\underline{\underline{J}}$ is non-zero.

(d) There is a unique solution to the system of linear equations $\underline{\underline{J}}\underline{x} = \underline{R}$ for any real nx1 vector, $\underline{R}$.

(e) The reduced row echelon form of $\underline{\underline{J}}$ will not have any rows completely filled with zeroes.

Problem 7. Solution of a Nonlinear Algebraic Equation

Consider the equation , $3x - \sqrt{x} = 2$. Using an initial guess of 3.0, complete one entire iteration of a Newton-Raphson step.

Problem 8. Solution of a system of Nonlinear Algebraic Equations

Consider the following steady-state mass and energy balance for a non-isothermal continuous stirred tank reactor with an irreversible first order reaction (Notation is the same as in the project.):

$$0 = \frac{F_{in}C_{A,in} - F_{out}C_A - C_A k_o e^{-\frac{E_a}{RT}}}{V}$$

$$0 = \frac{F_{in}\tilde{H}_{in} - F_{out}\tilde{H}_{out}(T) + \Delta H_r C_A k_o e^{-\frac{E_a}{RT}} + \dot{Q}}{V}$$

A multivariate Newton-Raphson method has proved successful in giving us the steady-state values of C_A and T for slightly exothermic reactions (a value of ΔH_r). However, when we attempt the same solution technique for a more exothermic reaction with (a values of $10\Delta H_r$), we cannot find an initial guess close enough to the steady state values of C_A and T to converge to our solution. What do you suggest we do? Give an algorithm and equations if necessary.

Problem 9. Numerical Integration

Perform a numerical integration using Trapezoidal rule on the function, $f(x) = 500x^2$ over the range $-2 \le x \le 0$ using 2 intervals.

Problem 10. Solutions of Ordinary Differential Equations

Perform one Euler method step on the ordinary differential equation: $\frac{d^2y}{dx^2} = \frac{dy}{dx}\sin(x)\cos(y)$ subject to the initial conditions, $y(x=1) = 2$ and $\frac{dy}{dx}(x=1) = 0.5$, using a step size $\Delta x = 0.1$ x and y are given in radians. (Determine the values, $y(x=1.1)$. and $\frac{dy}{dx}(x=1.1)$.)

Problem 11. Regression

We are trying to fit some data to $\hat{y} = bx + a$ We are given the following information:

$$\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y}) = 0.619 \qquad \sum_{i=1}^{n}(x_i - \bar{x})^2 = 6.32 \qquad n = 100$$

$$\sum_{i=1}^{n} y_i = 147.8 \qquad \sum_{i=1}^{n} x_i = 2509 \qquad SSE = 0.25$$

Calculate the (i) average slope, (ii) the standard deviation of the slope, and (iii) a 95% confidence interval on the slope.

Problem 12. ANOVA

Consider the problem where we test three catalysts for their effect on moles per minute of product generated in a reactor. We run the anova_1factor.m code and obtain this output:

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» anova_1factor
  Problem with    3 treatments and    6 replicates

y =[1.0000    1.1000    1.2000    1.1000    1.2000    1.0000
    2.0000    2.2000    2.4000    2.2000    2.4000    2.0000
    1.0000    1.1000    1.2000    1.1000    1.2000    1.0000]

  Ho: all treatments are equal
  Reject Ho if  151.25 >> f(    2,   15 )
  Hypothesis Rejected for 98 percent confidence interval (  151.25 >    6.36 )

  pvalue =   1.14e-010

  95 percent C.I. on the    1 treatment:  9.30e-001 <  1.10e+000 <  1.27e+000
  95 percent C.I. on the    2 treatment:  2.03e+000 <  2.20e+000 <  2.37e+000
  95 percent C.I. on the    3 treatment:  9.30e-001 <  1.10e+000 <  1.27e+000

  95 percent C.I. on the    1 -    2 treatment diff.: -1.34e+000 < -1.10e+000 < -8.60e-001
  95 percent C.I. on the    1 -    3 treatment diff.: -2.40e-001 <  0.00e+000 <  2.40e-001
  95 percent C.I. on the    2 -    3 treatment diff.:  8.60e-001 <  1.10e+000 <  1.34e+000
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Answer these questions:

(i) What does the treatment represent?
(ii) What does y represent?
(iii) The null hypothesis was rejected. What does this mean--do the different catalysts make a difference?
(iv) Rank the catalysts in order of best performance.
(v) Does any one catalyst give a yield of at least 1.0 moles/minute more product than every other catalyst, within a 95% confidence interval? If so, which one?
(vi) What does the reported p-value mean?