Exam II
Administered: Wednesday, March 1, 2000
28 points

For each problem part: 0 points if not attempted or no work shown,
1 point for partial credit, if work is shown,
2 points for correct numerical value of solution

Problem 1. (14 points)

We are investigating two different methods for synthesizing a chemical product. In the laboratory, the two methods have been used to generate samples, from which the mean and variance of the concentration (mol/liter) have been measured. Assume that the true population variances, while unknown, are equal.
Sample statistics

$$n_1 = 16 \qquad \overline{x}_1 = \frac{1}{16}\sum_{i=1}^{16} x_i = 2.280 \qquad s_1^{\,2} = \frac{1}{16}\sum_{i=1}^{16}\left[(x_i - \overline{x}_1)^2\right] = 0.0229 \qquad s_1 = 0.1513$$

$$n_2 = 10 \qquad \overline{x}_2 = \frac{1}{10}\sum_{i=1}^{10} x_i = 2.128 \qquad s_2^{\,2} = \frac{1}{10}\sum_{i=1}^{10}\left[(x_i - \overline{x}_2)^2\right] = 0.0744 \qquad s_2 = 0.2728$$

(a) What PDF is appropriate for determining a confidence interval on this difference of means?
(b) Find the lower limit on a 98% confidence interval on the difference of means.
(c) Find the upper limit on a 98% confidence interval on the difference of means.
(d) Based on this confidence interval, does method 1 yield a concentration at least 0.1 mol/*l* greater than method 2?
(e) What is the probability that method 1 yields a concentration at least 0.1 mol/liter greater than method 2?
(f) The two methods of synthesis in problem 1 have associated manufacturing costs of \$0.089 per liter for process 1 and \$0.060 per liter for process 2, given the same production rate of 100,000 liters per day. What are the average production costs per mole?
(g) Which is the better method based on economics?

Solution:

(a)-(d)
This problem requires the t-distribution because we want the distribution of the difference of sample means when the population variance is unknown.

$v = n_1 + n_2 - 2 = 24$ because population variances are equal. The t-value from the table is 2.492

$$P\left[\begin{array}{l}(2.280 - 2.128) - 2.492\sqrt{\dfrac{0.0229}{16} + \dfrac{0.0744}{10}} < \\ \qquad\qquad (\mu_1 - \mu_2) < (2.280 - 2.128) - 2.492\sqrt{\dfrac{0.0229}{16} + \dfrac{0.0744}{10}}\end{array}\right] = 0.98$$

$$P\left[-0.0827 < (\mu_1 - \mu_2) < 0.3867\right] = 0.98$$

The difference of population means of 0.1 does fall within a 95% confidence interval.

(e)

$$T = \frac{\left(\overline{X}_1 - \overline{X}_2\right) - \left(\mu_1 - \mu_2\right)}{\sqrt{\left(\frac{s_1^2}{n_1}\right) + \left(\frac{s_2^2}{n_2}\right)}} = \frac{\left(2.280 - 2.128\right) - 0.1}{\sqrt{\frac{0.0229}{16} + \frac{0.0744}{10}}} = 0.5521$$

From Table A.4 of WMM, for v = 24

$$P\left[\mu_1 - \mu_2 > 0.1\right] = P\left[t < 0.55\right] = 1 - P\left[t < 0.55\right] = 1 - 0.29 = 0.71$$

(f) we want the mean daily production costs:

$$g(x) = \left(C\frac{\$}{\text{liter}}\right) \Big/ \left(\overline{x}_i \frac{\text{mole}}{\text{liter}}\right) = \left(\frac{\text{cost}(\$)}{\text{mole}}\right)$$

$$\mu_{\text{cost},1} = \frac{0.089}{2.280} \cdot \overline{x}_1 = 0.0390 \frac{\$}{\text{mole}}$$

$$\mu_{\text{cost},2} = \frac{0.060}{2.128} \cdot \overline{x}_2 = 0.0282 \frac{\$}{\text{mole}}$$

(g) The second method is cheaper.

Problem 2. (6 points)

We manage a chemical plant. It has a probability of violating an EPA regulation on stack emissions of 0.002 on any given day. If we violate the regulation twice or more in 20 days, we must shut down the plant and undergo an inspection.
(a) If we violated the regulation on start up (day 1), what is the probability that the plant must be shut down before 19 more days have elapsed?
(b) If we manage not to violate the regulation on start up (day 1), what is the probability that the plant must be shut down before 19 more days have elapsed?
(c) We have an additional EPA regulation that we not exceed 4 violations per year. What is the probability that our plant shuts down during the course of 1 year (365 days)?

Solution:

(a) This requires the binomial distribution, where x=0, n=19, p=0.002

A shutdown means we have one or more additional violations in the remaining 19 days:

$$P(X \geq 1) = 1 - P(X < 1) = 1 - P(X = 0)$$

$$P(X=0)=b(0;19,0.002)=\binom{19}{0}0.002^{0}0.998^{19}=0.9627$$

$$P(X\geq 1)=1-0.9627=0.0373$$

(b) This requires the binomial distribution, where x=0, n=19, p=0.002

A shutdown means we have one or more additional violations in the remaining 19 days:

$$P(X\geq 2)=1-P(X\leq 1)=1-P(X=1)-P(X=0)$$

$$P(X=1)=b(1;19,0.002)=\binom{19}{1}0.002^{1}0.998^{18}=0.0367$$

$$P(X\geq 2)=1-P(X\leq 1)=1-0.0367-0.9627=0.0006$$

(c) N is large. Approximate binomial with Poisson.

t = 1 year. $\lambda t=np=365*0.002=0.73\approx 0.7$ violations/year. From table A.2:

$$P(X>4)=1-P(X\leq 4)=1-0.9992=0.0008$$

Problem 3. (4 points)

We have a pair of compressors (one primary and one back-up) on a service line. The compressors have functional life-times of 6 months before they need to be maintained. Maintenance takes 2 weeks.
(a) Select an appropriate continuous distribution to determine the probability that the back-up compressor fails while the primary compression is being maintained?
(b) Select an appropriate discrete distribution to determine the probability that the back-up compressor fails while the primary compression is being maintained?

Solution:

(a) With only a mean, we can assume this is an exponential distribution.

$\beta=\mu=6$ We want: $P(t<0.5)$

$$P(t<0.5)=\int_0^{0.5}f(x)dx=\int_0^{0.5}\frac{1}{\beta}e^{-x/\beta}dx=1-e^{-0.5/6}=0.0800$$

(b) Or , we can assume this is a Poisson distribution.

x = number of failures

t = 0.5 months. $\lambda=\frac{1}{6}\frac{\text{failures}}{\text{month}}$ $\lambda t=\frac{1}{12}\approx 0.1$ From table A.2:

$$P(X\geq 0)=1-P(X=0)=1-0.9048=0.0952$$

Problem 4. (4 points)

The plastic rings which hold six-packs of soda together have to have sufficient mechanical strength to hold the cans in together, but not so great strength that it becomes difficult to remove the cans from the plastic rings. In studying what the proper mechanical strength of a new plastic material, proposed for this purpose, we are told that the material has a mean strength of 0.1 Newton and a variance of 0.0004 Newtons^2.
(a) What fraction of plastic rings from this material would release a 0.1 kg can of soda being tugged from the plastic ring with an acceleration of 0.5 m/s^2?
(b) What strength of material would correspond to 5% of the plastic rings releasing under the conditions in (a)?

Solution:

(a) This requires the normal distribution. $\mu = 0.1\ \sigma = 0.02$

$$P(x \le 0.05) = P\left(z \le \frac{0.05 - 0.1}{0.02}\right) = P(z \le -2.5) = 0.0062$$

(b) This requires the normal distribution. $\mu = 0.1\ \sigma = 0.02$

$P(Z \le z) = 0.05$ From table A.3 of WMM, $z = -1.645$

$z = \frac{x - \mu}{\sigma}$, then $x = \mu + \sigma z = 0.1 + 0.02 * -1.645 = 0.0671$