Applied Statistics and Numerical Methods for Engineers
ChE 301, Fall 1998
Midterm Exam Number One
Administered: Monday, September 21, 1998

In all relevant problems: WRITE DOWN THE FORMULA YOU USE, BEFORE YOU USE IT.
In all problems: NO NUMERICAL INTEGRATION REQUIRED.
ALL PROBLEMS ARE WORTH 2 POINTS. THE EXAM HAS 36 POINTS.

For problems (1.1) to (1.7), consider that you have a group of 6 engineers, named Ali, Barbara, Charlie, Denise, Ernie, and Feng. Ali and Barbara are environmental engineers. Charlie and Denise are material scientists. Ernie and Feng are chemical engineers. They are to be randomly split up into 2 groups of three people.

(1.1) Define the sample space. (Either with words or lists.) Define one example of an event from that sample space.

(1.2) What are the number of ways that 2 groups of three can be formed, irrespective of engineering discipline?

(1.3) Consider the number of ways that 2 groups can be formed, considering the 6 people only by their profession?

Possible outcomes of a group	Number of ways for each outcome
2 Environmental, 1 Chemical, 0 Materials	2
2 Environmental, 0 Chemical, 1 Materials	2
1 Environmental, 2 Chemical, 0 Materials	2
0 Environmental, 2 Chemical, 1 Materials	2
0 Environmental, 1 Chemical, 2 Materials	2
1 Environmental, 0 Chemical, 2 Materials	2
1 Environmental, 1 Chemical, 1 Materials	X

Find X.

(1.4) What is the probability of having the 2 chemical engineers in different groups?

(1.5) What is the probability of the 2 chemical engineers in different groups OR the 2 materials scientists in the same group?

(1.6) What is the probability of having the 2 materials scientists in the same group GIVEN the 2 chemical engineers are in different groups.

(1.7) Determine rigorously whether having the 2 materials scientists in the same group and having the 2 chemical engineers in different groups are independent.

(2.1) State the three conditions that must be satisfied in order for a continuous probability density to be valid.

(2.2) For the PDF $f(x) = \begin{cases} \frac{x}{16} & \text{for } 2 \le x \le 6 \\ 0 & \text{otherwise} \end{cases}$, find $P(X \ge 4)$

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(2.3) For the PDF $f(x) = \begin{cases} 0.125 & \text{for } x = 2 \\ 0.375 & \text{for } x = 4 \\ 0.25 & \text{for } x = 6 \\ 0.25 & \text{for } x = 8 \end{cases}$, find $P(2 < X \leq 6)$

(2.4) Which of the following statements are TRUE?

(a) A histogram is a plot of the cumulative PDF.
(b) The cumulative PDF monotonically increases.
(c) A symmetric PDF gives rise to a symmetric cumulative PDF.
(d) The continuous cumulative PDF, $F(x)$, has a value of 0.5 when $x = \mu_x$.

For Problems (3.1) to (3.6) Consider the following: x and y are random variables. d and p are functions of x and y,
d = y - x and p = xy. 100 random samples of x and y values are taken. The following data is then calculated for those 100 samples. (Not all of the raw data is shown below.)

run	x	y	d	p	x^2	y^2	d^2	p^2	xd	xy	yp
1	10.90	18.74	7.85	204.30	118.79	351.37	61.55	41739.44	85.51	186.69	3829.59
2	11.17	19.73	8.55	220.36	124.80	389.08	73.17	48556.36	95.56	172.26	4346.53
3	9.34	20.14	10.80	188.14	87.23	405.77	116.72	35395.82	100.91	239.70	3789.78
…	…	…	…	…	…	…	…	…	…	…	…
98	11.32	18.41	7.09	208.43	128.20	338.87	50.21	43443.02	80.23	176.79	3836.85
99	10.57	21.80	11.23	230.47	111.79	475.18	126.01	53118.67	118.69	235.07	5024.02
100	11.69	19.66	7.97	229.70	136.56	386.37	63.53	52763.36	93.14	211.16	4515.11
sum	1013.00	1979.94	XXXX	20041.70	10381.3	39327.6	XXXX	4070754.6	9660.4	19897.98	397789.3
mean	10.13	19.80	XXXX	200.42	103.81	393.28	XXXX	40707.55	96.60	198.98	3977.89

(3.1) Find the variance of x, σ_x^2.

(3.2) Find the standard deviation of y, σ_y.

(3.3) Find the mean of d, μ_d.

(3.4) Find the variance, σ_d^2, of d, <u>assuming x and y are independent.</u>

(3.5) Find the covariance of x and d, σ_{xd}.

(3.6) Find the correlation coefficient, ρ_{xd}.

(3.7) Explain the sign of ρ_{xd}.