Exam II
Administered: Wednesday, October 9, 2002
? points

For each problem part: 0 points if not attempted or no work shown,
1 point for partial credit, if work is shown,
2 points for correct numerical value of solution

Problem 1. (10 points)

We are using a CDR burner to burn 700 MB cds. Sometimes, the cds contain defective sectors. We burn 20 CDRs. From our sample, we find a sample mean of defective bytes of 5.8 MB with a sample standard deviation of 1.2 MB.

Based on this information, answer the following questions.

(a) What PDF is appropriate for determining a confidence interval on the mean in this problem?
(b) Find the lower limit on a 95% confidence interval on the mean.
(c) Find the upper limit on a 95% confidence interval on the mean.
(d) Are we 95% confident that this CDR burner creates cds with less than 1 MB of defective data?
(e) What fraction of CDR burners create cds with less than 2 MB of defective data?

Solution:

(a) What PDF is appropriate for determining a confidence interval on the mean in this problem?

We must use the t-distribution to determine the confidence interval on the mean when the population variance is unknown.

(b) Find the lower limit on a 95% confidence interval on the mean.
(c) Find the upper limit on a 95% confidence interval on the mean.

$$\bar{x} = 5.8 \qquad n = 20 \qquad s = 1.2$$

$$\alpha = \frac{1 - C.I.}{2} = 0.025$$

$$v = n - 1 = 19$$

$$t_\alpha(v) = t_{0.025}(19) = 2.093$$ from Table A.4 of WMM

$$t_{1-\alpha} = -t_\alpha$$

confidence interval:

$$P\left[\overline{X}_1 + t_{1-\alpha}\sqrt{\frac{s_1^2}{n_1}} < \mu < \overline{X}_1 + t_\alpha\sqrt{\frac{s_1^2}{n_1}}\right] = 1 - 2\alpha$$

$$P[5.2384 < \mu < 6.3616] = 0.95$$

(d) Are we 95% confident that this CDR burner creates cds with less than 1 MB of defective data?

(e) What fraction of CDR burners create cds with less than 2 MB of defective data?

This is the reverse problem. This is the given-t, find p problem.

$$T_{lo} = \frac{x_{lo} - \mu}{\sqrt{\left(\frac{s^2}{n}\right)}} = \frac{2 - 5.8}{\sqrt{\left(\frac{1.44}{20}\right)}} = \text{-}14.16$$

The table in Appendix A.4 does not contain negative values for T. However, due to the symmetry of the T-distribution, we can see that

$$P(t < T_{lo}; v = 19) = P(t > -T_{lo}; v = 19)$$

The table does not contain values of t as high as 14.16. Therefore, the best we can say for this part of the problem is that the probability has to be less than the smallest probability given in the table, which is 0.0005

$$P(t < T_{lo}; v = 19) < 0.0005$$

Problem 2. (8 points)

A particular device is powered by four batteries. Each battery has a mean life time of 9 months. The device only operates if all four batteries continue to function.

(a) What PDF would describe the probability that an individual battery is operating after ten months?
(b) What is the probability that an individual battery is operating after ten months?
(c) What PDF would describe the probability that all 4 batteries are functioning after ten months?
(d) What is the probability that the device still functions after ten months?

Solution:

(a) What PDF would describe the probability that an individual battery is operating after ten months?

The exponential PDF would describe the life-time of a single battery.

(b) What is the probability that an individual battery is operating after ten months?

$$P(t > 10) = \int_{10}^{\infty} f_e(t;\beta)dt = \int_{10}^{\infty} \frac{1}{\beta} e^{-\frac{t}{\beta}} dt = -e^{-\frac{t}{\beta}}\Bigg|_{10}^{\infty} = -e^{-\frac{\infty}{\beta}} - -e^{-\frac{10}{\beta}} = 0 + e^{-\frac{10}{\beta}}$$

$$P(t > 10) = e^{-\frac{10}{9}} = 0.3292$$

(c) What PDF would describe the probability that all 4 batteries are functioning after ten months?

This calls for the binomial probability because the batteries are independent and all have the same probability of operating after ten months have passed.

(d) What is the probability that the device still functions after ten months?

x = random variable = number of batteries operating after ten months = 4
n = total number of batteries = 4
p = probability that an individual battery is operating after ten months = answer to part (b)

$$P(X = x) = b(x;n,p) = \binom{n}{x} p^x q^{n-x}$$

$$P(x = 4) = b(4;4,0.3292) = 0.0117$$

Problem 3. (8 points)

We run a warranty company that provides replacement parts for digital cameras. If our research team tells us that on average digital cameras have a lifetime of 6 years with a standard deviation of 1.5 years, then answer the following questions.

(a) If we provide a warranty for all cameras lasting less than 4 years, what fraction of the cameras can we expect to replace?

(b) If we only want to replace 2% of the cameras, how long should our warranty last?

(c) What PDF did you use to solve (a) & (b)?

(d) If we want our warranty program simply to break even, and the average cost of a digital camera is $300. How much should we charge for the 4-year (part a) warranty protection?

Solution:

(a) If we provide a warranty for all cameras lasting less than 4 years, what fraction of the cameras can we expect to replace?

$$P(x < 4) = P(z < \frac{x-\mu}{\sigma}) = P(z < \frac{4-6}{1.5}) = P(z < -1.3333) = 0.0918$$

From table A.3.
We can expect 9.18 percent of the cameras to fail before one year.

(b) If we only want to replace 2% of the cameras, how long should our warranty last?

$$P(z < z_{lo}) = 0.02$$

$$z_{lo} = -2.054 \quad \text{from table A.3}$$

$$z_{lo} = \frac{x_{lo} - \mu}{\sigma}$$

$$x_{lo} = \mu + \sigma z_{lo} = 6 + 1.5(-2.054) = 2.919 \text{ years}$$

Our warranty should last 2.919 years

(c) What PDF did you use to solve (a) & (b)?

Normal distribution. The variable is continuous and we have only been provided the mean and the standard deviation.

(d) If we want our warranty program simply to break even, and the average cost of a digital camera is $300. How much should we charge for the 4-year (part a) warranty protection?

Since a camera costs $300 and 9.18% of the cameras fail, we should charge $27.54 in order to break even.