Exam I
Administered: Monday, September 16, 2001
20 points

For each problem part: 0 points if not attempted or no work shown,
1 point for partial credit, if work is shown,
2 points for correct numerical value of solution

Problem 1. (16 points)

A railroad tanker containing concentrated sulfuric acid derails near a populated area. The concentration of sulfuric acid in the air as a function of the radial position from the point of the derailment at its worst time is given by the following function, where r is in miles, and c is in ppm

$$c(r) = \begin{cases} 20\exp(-2r) & \text{for } 0 \le r \le 10 \\ 0 & \text{otherwise} \end{cases}$$

The probability distribution of the concentration of sulfuric acid is proportional to the concentration.

$$f(r) = 2\pi c_o c(r) r$$

(a) What is the random variable in this problem, both in terms of physical interpretation and the variable used?
(b) What value of c_o will make this PDF a legitimate function?
(c) What fraction of the sulfuric acid is located within 1 mile of the derailment?
(d) What fraction of the sulfuric acid is located beyond 1 mile of the derailment?
(e) What is the mean sulfuric acid concentration in the ten mile radius?

You may find the following indefinite integrals useful:

$$\int ar\exp(-br)\,dr = -\frac{a}{b^2}(br+1)\exp(-br)$$

$$\int ar^2\exp(-br)\,dr = -\frac{a}{b^3}\left(b^2r^2+2br+2\right)\exp(-br)$$

$$\int ar^3\exp(-br)\,dr = -\frac{a}{b^4}\left(b^3r^3+3b^2r^2+6br+6\right)\exp(-br)$$

Solution:

(a) What is the random variable in this problem, both in terms of physical interpretation and the variable used?

The random variable, r, is the radial position. c(r) is a function of the random variable r.

(b) What value of c_o will make this PDF a legitimate function?

$$P(-\infty < x < \infty) = 1 = \int_{-\infty}^{\infty} f(r)dr = \int_{0}^{10} 2\pi c_o c(r) r dr = \int_{0}^{10} 2\pi c_o 20\exp(-2r) r dr$$

$$c_o = \frac{1}{2\pi 20 \int_0^{10} \exp(-2r) r dr} = \frac{1}{2\pi 20\left[-\frac{1}{2^2}(2r+1)\exp(-2r)\right]_0^{10}}$$

$$c_o = \frac{1}{2\pi 20\left[\left(-\frac{1}{2^2}(2\cdot 10+1)\exp(-2\cdot 10)\right)-\left(-\frac{1}{2^2}\right)\right]} \approx 0.0318$$

(c) What fraction of the sulfuric acid is located within 1 mile of the derailment?

$$P(0 < x < 1) = \int_0^1 f(r)dr = \int_0^1 2\pi c_o 20\exp(-2r) r dr = 2\pi 20 c_o\left[-\frac{1}{2^2}(2r+1)\exp(-2r)\right]_0^1$$

$$P(0 < x < 1) = 2\pi 20 c_o\left[\left(-\frac{1}{2^2}(2\cdot 1+1)\exp(-2\cdot 1)\right)-\left(-\frac{1}{2^2}\right)\right] \approx 0.5940$$

(d) What fraction of the sulfuric acid is located beyond 1 mile of the derailment?

$$P(x > 1) = 1 - P(0 < x < 1) \approx 1 - 0.5940 = 0.4060$$

(e) What is the mean sulfuric acid concentration in the ten mile radius?

$$\mu_c = \int_{-\infty}^{\infty} c(r) f(r) dr = \int_0^{10} c(r) 2\pi c_o c(r) r dr = \int_0^{10} (20\exp(-2r)) 2\pi c_o 20\exp(-2r) r dr$$

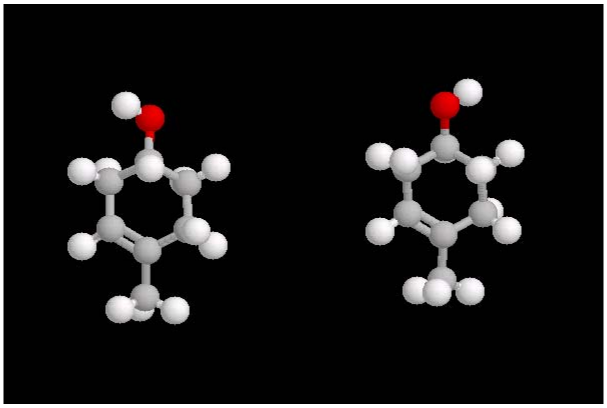
$$\mu_c = \pi c_o 800 \int_0^{10} \exp(-4r) r dr = \pi c_o 800 \int_0^{10} \exp(-4r) r dr$$

$$\mu_c = \pi c_o 800\left[-\frac{a}{b^2}(br+1)\exp(-br)\right]_0^{10} = \pi c_o 800\left[-\frac{1}{16^2}(4r+1)\exp(-4r)\right]_0^{10}$$

$$\mu_c = \pi c_o 800\left[\left(-\frac{1}{4^2}(4r+1)\exp(-4r)\right)-\left(-\frac{1}{4^2}\right)\right]_0^{10} \approx 5.000 ppm$$

Problem 2. (10 points)

Seudenol, $C_7H_{12}O$, is an aggregation pheromone from the female Douglas fir beetle, *Dendroctonus pseudotsugae*. The natural pheromone is a racemic mixture which is much more biologically active than either single enantiomer. The two enatiomers, (R)-seudenol and (S)-seudenol, are shown in the figure below.



(R)-Seudenol (S)-Seudenol

We are studying two alternative methods, method A and method B to synthesize this mixture. Method A was used to generate 40% of the product. Method A produced 64% (R)-seudenol. Method B produces 76% (R)-seudenol. Answer the following questions. Where appropriate, report to 4 significant figures.

(a) Draw a Venn Diagram of the sample space for the process and classification of the molecules in the product.
(b) What is the probability that a molecule was synthesize using method A and is (R)-seudenol?
(c) What is the probability that a molecule is (R)-seudenol?
(d) What is the probability that a molecule was generated using method B given that it is (R)-seudenol?
(e) What is the probability that a molecule was synthesize using method B and is (S)-seudenol?

Solution:

We are given:

$$P(A) = 0.40$$
$$P(R \mid A) = 0.64$$
$$P(R \mid B) = 0.76$$

(a) Draw a Venn Diagram of the sample space for the process and classification of the product.

$A \cap R$	$B \cap R$
$A \cap S$	$B \cap S$

(b) What is the probability that a molecule was synthesize using method A and is (R)-seudenol?

$$P(R \mid A) = \frac{P(R \cap A)}{P(A)}$$

$$P(R \cap A) = P(R \mid A)P(A) = 0.64 \cdot 0.40 = 0.256$$

(c) What is the probability that a molecule is (R)-seudenol?

$$P(B) = 1 - P(A)$$

$$P(R \mid B) = \frac{P(R \cap B)}{P(B)}$$

$$P(R \cap B) = P(R \mid B)P(B) = P(R \mid B)\left[1 - P(A)\right] = 0.76 \cdot \left[1 - 0.40\right] = 0.456$$

$$\begin{aligned} P(R) = P\left[(R \cap A) \cup (R \cap B)\right] &= P(R \cap A) + P(R \cap B) - P\left[(R \cap A) \cap (R \cap B)\right] \\ &= 0.256 \quad + 0.456 \quad - 0 \qquad = 0.712 \end{aligned}$$

(d) What is the probability that a molecule was generated using method B given that it is (R)-seudenol?

$$P(B \mid R) = \frac{P(R \cap B)}{P(R)} = \frac{0.456}{0.712} = 0.6404$$

(e) What is the probability that a molecule was synthesize using method B and is (S)-seudenol?

$$P(B) = P\left[(S \cap B) \cup (R \cap B)\right] = P(S \cap B) + P(R \cap B) - P\left[(S \cap B) \cap (R \cap B)\right]$$

$$P\left[(S \cap B) \cap (R \cap B)\right] = 0$$

$$P(S \cap B) = P(B) - P(R \cap B) = 0.6 - 0.456 = 0.144$$