Exam IV: Administered: Monday December 10, 2001

Problem (1) (10 points)
Perform one complete Newton-Raphson iteration on the system of equations:

$$x + y = 4 \qquad\qquad xy = 2$$

Use (x,y) = (1/2,7/2) as your initial guess.
Along the way, present the Jacobian, Residual, determinant, inverse, and new estimate of [x,y].

Problem (2). (14 points)

Consider the linear ordinary differential equation initial value problem

$$\frac{dy}{dx} = 3x^2 \tag{2.1}$$

where $y(x_o = 1) = y_o = 1$

The analytical solution to this ODE IVP can be obtained by separating variables:

$$dy = 3x^2 dx \tag{2.2}$$

$$\int_{y_o}^{y} dy = \int_{x_o}^{x} 3x^2 dx \tag{2.3}$$

$$y(x) - y_o = \int_{x_o}^{x} 3x^2 dx \tag{2.4}$$

$$y(x) = y_o + \int_{x_o}^{x} 3x^2 dx \tag{2.5}$$

$$y(x) = y_o + x^3 - x_o^3 \tag{2.6}$$

(a) Solve the ODE as it is given in equation (2.1) using the Euler Method. Use a step size of $\Delta x=1$ and report the solution at x=3.
(b) Solve the ODE as it is given in equation (2.5) using the Trapezoidal Rule. Again, use a step size of $\Delta x=1$ and report the solution at x=3.
(c) Report the analytical solution of the ODE at x=3.
(d) Which method is more accurate in this problem, the Euler Method or the Trapezoidal method?
(e) Why?

Problem (3) (8 points)

In solving the solution to $\underline{\underline{A}}\underline{x} = \underline{b}$, where $\underline{\underline{A}} = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 2 \\ 4 & 3 & 4 \end{bmatrix}$ and $\underline{b} = \begin{bmatrix} 3 \\ 5 \\ 12 \end{bmatrix}$, we find the following information on the reduced row echelon form of the A|b augmented matrix.

$$\text{rref}\left(\underline{\underline{A}} \mid \underline{b}\right) = \left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

(a) What is the determinant of A?
(b) Does the inverse of A exist?
(c) How many solutions exist to $\underline{\underline{A}}\underline{x} = \underline{b}$?
(d) If infinite solutions exist, find the solution $\underline{x} = \begin{bmatrix} x_1 \\ x_2 \\ 1 \end{bmatrix}$.

Problem (4) (8 points)

You are sampling from a reactor that is creating polymer with an average molecular weight of 7.5×10^6 amu and a standard deviation of 3.0×10^6 amu.

(a) What fraction of the product has a molecular weight less than 4.5×10^6 amu ?
(b) What PDF did you use to solve part (a)?
(c) 90% of the product has a molecular weight greater than what value?
(d) What PDF did you use to solve part (c)?

Problem 5. (12 points)

A manufacturer of automobile batteries claims that their batteries last on average 8 years. You buy two batteries. The first fails in 6 years and the second fails in 3 years.

(a) What is the sample mean of the battery life?
(b) What is the sample standard deviation?
(c) What is the lower limit on a 95% confidence interval for the mean battery life?
(d) What is the upper limit on a 95% confidence interval for the mean battery life?
(e) Is the manufacturer's claim reasonable?
(f) Explain your answer to part (e).

Problem (6) (4 points)

In Computer Project 2, you solved the steady state behavior of the reactor in the adiabatic and nonadiabatic mode.

(a) Which mode yielded a higher steady state operating temperature in the reactor?
(b) Why?