Exam I
Administered: Monday, September 17, 2001
26 points

For each problem part: 0 points if not attempted or no work shown,
1 point for partial credit, if work is shown,
2 points for correct numerical value of solution

Problem 1. (16 points)

The gender of alligators depends upon the temperature, T, at which they incubate inside the nest. Scientists have determined that the probability that a fraction of eggs in an alligator nest that turn out male, x, obeys the probability function

$$f(x;T) = \begin{cases} \dfrac{x^2 - 2\dfrac{(T-70)}{50}x + 1}{\dfrac{4}{3} - 2\dfrac{(T-70)}{100}} & \text{for } 0 \le x \le 1, \text{and } 70 \le T \le 120\ ^\circ\text{F} \\ 0 & \text{otherwise} \end{cases}$$

(a) What is the random variable in this problem, both in terms of physical interpretation and the variable used?
(b) What is the probability that more than 75% of the eggs hatch male at 80 F?
(c) What is the probability that less than 75% of the eggs hatch male at 80 F?
(d) What is the average fraction of male eggs at 80 F?
(e) What is the average fraction of male eggs at 100 F?
(f) What is the average fraction of male eggs at 150 F?
(g) What is the average fraction of female eggs at 80 F?
(h) Are female eggs favored at low or high temperature? Why?

Problem 2. (10 points)

You are performing a lab-scale feasibility study for production of a pharmaceutical using either a batch or a continuous process. If the product doesn't meet certain purity specifications, it is deemed defective, rather than good. The batch process was used 64% of the time. The probability that a sample of the product is defective and created by the batch process is 0.04. The probability that a sample is good given that it was created by the continuous process is 0.97. Answer the following questions. Where appropriate, report to 4 significant figures.

(a) Draw a Venn Diagram of the sample space for the process and classification of the product.
(b) What is the probability that a sample is defective given that it was produced from the batch process?
(c) What is the probability that a sample was produced from the continuous process?
(d) What is the probability that a sample is good and was produced by the continuous process?
(e) What is the probability that the product is defective?