Homework Assignment Number Eight
Assigned: Wednesday, March 17, 1999
Due: Wednesday, March 24, 1999 BEFORE LECTURE STARTS.

Problem 1. Geankoplis 4.1-2, page 317

$$\frac{q_z}{A} = -k\frac{(T_2 - T_1)}{(z_2 - z_1)} = -k\frac{\Delta T}{\Delta z}$$

$$35.1 = k\frac{(318.4 - 303.2)}{(0.025)}$$

$$k = 0.0577\frac{W}{m \cdot K} = 0.0333\frac{btu}{hr \cdot ft \cdot F}$$

Problem 2. Geankoplis 4.2-1, page 317
For steady state, constant area, and a thermal diffusivity that is a linear function of temperature diffusing in the radial direction of a cylinder

$$\frac{q_z}{A} = -k\frac{dT}{dz}$$

$$A(r) = 2\pi rL \text{ and } k(T) = a + bT$$

$$\int_{r_1}^{r_2}\frac{q_r}{A}dr = \int_{r_1}^{r_2}\frac{q_r}{2\pi rL}dr = \frac{q_r}{2\pi L}\int_{r_1}^{r_2}\frac{1}{r}dr = -\int_{T_1}^{T_2}kdT = -\int_{T_1}^{T_2}(a + bT)dT$$

$$\frac{q_r}{2\pi L}\ln\left(\frac{r_2}{r_1}\right) = -a(T_2 - T_1) - \frac{b}{2}\left(T_2^2 - T_1^2\right)$$

$$q_r = -2\pi L\frac{a(T_2 - T_1) - \frac{b}{2}\left(T_2^2 - T_1^2\right)}{\ln\left(\frac{r_2}{r_1}\right)}$$

You can rewrite this as:

$$q_r = -k_m A_{lm}\frac{\Delta T}{\Delta r}$$

where we define the log mean area to be:

$$A_{lm} = \frac{A_2 - A_1}{\ln\left(\frac{A_2}{A_1}\right)} = \frac{2\pi L(r_2 - r_1)}{\ln\left(\frac{r_2}{r_1}\right)}$$

and where the mean thermal conductivity is defined as:

$$k_m = a + \frac{b}{2}(T_2 + T_1)$$

Problem 3. Geankoplis 4.3-3, page 318

$$q = UA\Delta T$$

$$\Delta T = 27.8K, \quad A = 0.914m \cdot 1.83m = 1.67m^2$$

$$U = \frac{1}{\frac{\Delta x_A}{k_A} + \frac{\Delta x_B}{k_B} + \frac{\Delta x_C}{k_C}} = 3.863 \frac{W}{m^2 \cdot K}$$

$$q = 179.4W$$

Problem 4. Geankoplis 4.3-4, page 318

Steam condensing inside pipe. Ignore film coefficient.

$$L = 30.5m, \ \Delta T = 121.1 - 26.7 = 94.4K$$

$$D_1 = 0.0525m, \ D_2 = 0.06032m, \ D_3 = 0.11112m$$

$$\Delta r_S = 0.00391m, \ \Delta r_A = 0.0254m$$

$$A_{lm,S} = \frac{\pi L(D_2 - D_1)}{\ln\left(\frac{D_2}{D_1}\right)} = 5.396m^2, \ A_{lm,A} = \frac{\pi L(D_3 - D_2)}{\ln\left(\frac{D_3}{D_2}\right)} = 7.967m^2$$

$$k_S = 45\frac{W}{m \cdot K}, \ k_A = 0.182\frac{W}{m \cdot K}$$

$$q_r = -\frac{\Delta T}{\frac{\Delta r_S}{k_S A_{S,lm}} + \frac{\Delta r_A}{k_A A_{A,lm}}} = 5384W$$

$$q_r = \Delta\hat{H}_{vap}\dot{m},\ \Delta\hat{H}_{vap} = 2200\frac{kJ}{kg},\ \dot{m} = 31680\frac{kg}{s} = 8.8\frac{kg}{hr}$$

Problem 5. Geankoplis 4.3-10, page 319

$$q = UA\Delta T$$

$$\Delta T = 27.8K,\quad A = 0.914m \cdot 1.83m = 1.67m^2$$

$$U = \frac{1}{\frac{1}{h_{air}} + \frac{\Delta x_A}{k_A} + \frac{\Delta x_B}{k_B} + \frac{\Delta x_C}{k_C} + \frac{1}{h_{air}}} = 2.29\frac{W}{m^2 \cdot K}$$

$$q = 106.7W$$

Problem 6. Geankoplis 4.3-11, page 319

$$T(r) = T_w + \frac{\dot{q}}{4k}\left(R^2 - r^2\right)$$

$$T(r=0) = T_w + \frac{\dot{q}}{4k}\left(R^2\right) = 311.112K$$

Problem 7. Geankoplis 4.3-14, page 320

(a) $r_{crit} = \frac{k}{h_o} = \frac{0.08}{30} = 0.00267m$

$$L = 1m,\ \Delta T = 400 - 300 = 100K$$

$$D_1 = 0.030m,\ D_2 = 0.070m,\ \Delta r_l = 0.020m$$

$$A_{lm,l} = \frac{\pi L\left(D_2 - D_1\right)}{\ln\left(\frac{D_2}{D_1}\right)} = 0.1483m^2,$$

$$A_1 = \pi L D_1 = 0.0942m^2,\ A_2 = \pi L D_2 = 0.220m^2$$

$$k_l = 0.08\frac{W}{m \cdot K},\ h = 30\frac{W}{m^2 \cdot K}$$

(a) pipe without insulation

$$q_r = -\frac{\Delta T}{\frac{1}{hA_1}} = 2826\,W$$

(b) pipe with insulation

$$q_r = -\frac{\Delta T}{\frac{\Delta r_I}{k_I A_{I,lm}} + \frac{1}{hA_2}} = 54.4W$$