Homework Assignment Number Seven
Assigned: Wednesday, February 24, 1999
Due: Wednesday, March 3, 1999 BEFORE LECTURE STARTS.

Problem 1. Geankoplis, problem 3.3-1, page 207

Calculate brake hp of the pump. water, $\rho = 62.4 \frac{lb_m}{ft^3}$, $q = 60 \frac{gal}{min}$

$$\dot{m} = \rho q = 62.4 \frac{lb_m}{ft^3} 60 \frac{gal}{min} \frac{min}{60\,sec} \frac{ft^3}{7.481\,gal} = 8.341 \frac{lb_m}{sec}$$

At 60 gal/min, $\eta = 0.58$ and $head = 31 ft$ from Figure 3.3-2 page 136 Geankoplis.

The shaft work is $-W_s = H \frac{g}{g_c} = 31 \frac{lb_f \cdot ft}{lb_m}$ (3.3-4)

brake horse power is $brake\ hp = \frac{-W_s \dot{m}}{\eta 550} = 0.81 hp$ (3.3-2)

The plot on page 136 gives a brake horsepower of about 0.8 hp so it checks.

(b) repeat for $\rho = 0.85 \frac{g}{cm^3} = 53.1 \frac{lb_m}{ft^3}$

$$\dot{m} = \rho q = 53.1 \frac{lb_m}{ft^3} 60 \frac{gal}{min} \frac{min}{60\,sec} \frac{ft^3}{7.481\,gal} = 7.10 \frac{lb_m}{sec}$$

brake horse power is $brake\ hp = \frac{-W_s \dot{m}}{\eta 550} = 0.69 hp$

The plot on page 136 gives a brake horsepower of about 0.8 hp so it doesn't check well, which makes sense since the plot was made for water.

Problem 2. Geankoplis, problem 3.3-3, page 207

Adiabatic compression of air

$T_1 = 29.4C = 302.6K$, $q = 2.83 \frac{m^3}{min} = 0.0472 \frac{m^3}{s}$

$p_1 = 102.7 \frac{kN}{m^2} = 102700 Pa$, $p_2 = 311600 Pa$, $\eta = 0.75$

$MW = 29 \frac{gram}{mol} = 0.029 \frac{kg}{mol}$, $\gamma = 1.4$, $\rho = \frac{P \cdot MW}{RT} = 1.17 \frac{kg}{m^3}$

$$\dot{m} = \rho q = 1.17\frac{kg}{m^3}0.0472\frac{m^3}{s} = 0.0551\frac{kg}{s}$$

Find the power

$$-\hat{W}_s = \frac{\gamma}{\gamma-1}\frac{RT_1}{MW}\left[\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} - 1\right] = 113400\frac{J}{kg} \qquad (3.3\text{-}14)$$

$$\dot{W}_s = \hat{W}_s\dot{m} = -5352\frac{J}{s} = -6.25kW$$

$$\dot{W}_p = \frac{-\hat{W}_s}{\eta} = 8.33kW$$

Use adiabatic ideal gas law to calculate outlet temperature:

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} \text{ so } T_2 = 415.5K$$

Problem 3. Geankoplis, problem 3.4-5, page 208

Design an agitation system: $\rho = 950\frac{kg}{m^3}$, $\mu = 0.005\frac{kg}{m \cdot s}$, $V = 1.5m^3$

standard six blade open turbine with blades at 45 degree angles (curve 3, page 3.4-4)

$$\frac{D_a}{W} = 8, \ \frac{D_a}{D_t} = 0.35, \ \frac{P}{V} = 0.5\frac{kW}{m^3}$$

Find power

$$P = \frac{P}{V}V = 0.5\frac{kW}{m^3}1.5m^3 = 0.75kW$$

Find dimensions

Assume cylindrical tank

$$V = H\frac{\pi}{4}D_t^2$$

from table 3.4-1, page 144, Geankoplis: $\frac{H}{D_t} = 1$

$V = \frac{\pi}{4} D_t^3$ so $D_t = H = 1.24\text{m}$ tank diameter and tank height

$\frac{D_a}{D_t} = 0.35$ so $D_a = 0.43$ impeller diameter

$\frac{D_a}{W} = 8$ so , $W = 0.05$ impeller width in axial direction

$\frac{C}{D_t} = 0.33$ so $C = 0.41$ space between bottom of impeller and bottom of tank

$\frac{D_d}{D_a} = 0.67$ so $D_d = 0.29$ another diameter of impeller

$\frac{L}{D_a} = 0.25$ so $L = 0.11$ length of turbine blade in radial direction

$\frac{J}{D_t} = \frac{1}{12}$ so $J = 0.10$ width of baffle in radial direction

Find frequency:

Guess frequency: $N = 3 \frac{\text{rev}}{\text{sec}}$

$N_{Re} = \frac{D_a^2 N \rho}{\mu} = 105000$ and $N_P = \frac{P}{D_a^5 N^3 \rho} = 2.0$

Check Figure 3.4-4 for consistency. When $N_{Re} = 105000$, $N_P = 1.5$

Get new P, $N = \sqrt[3]{\frac{P}{D_a^5 \rho} \frac{1}{N_P}} = \sqrt[3]{\frac{53.7}{N_P}} = \sqrt[3]{\frac{53.7}{1.5}} = 3.3$

$N_{Re} = \frac{D_a^2 N \rho}{\mu} = 115500$

Check Figure 3.4-4 for consistency. When $N_{Re} = 105000$, $N_P = 1.5$

So $N = 3.3 \frac{\text{rev}}{\text{sec}}$

Problem 4. Geankoplis, problem 3.5-2, page 208

Pressure drop of pseudo-plastic fluid

ater, $\rho = 63.2 \frac{lb_m}{ft^3}$, $L = 100 ft$, $D = 2.067 in = 0.17225 ft$,

$\bar{v} = 0.500 \frac{ft}{s}$, $K = 0.280 \frac{lb_f \cdot s^n}{ft^2}$, $n = 0.50$

Generalized Reynolds number:

$$N_{Re} = \frac{D^{n'} \bar{v}^{2-n'} \rho}{g_c K \left(\frac{3n'+1}{4n'} \right)^{n'} 8^{n'-1}} = \frac{0.17225^{0.5} 0.5^{1.5} 63.2}{32.2 \cdot 0.280 \cdot 8^{-0.5}} = 2.60$$

so flow is laminar

$$f = \frac{16}{N_{Re}} = 6.15$$

$$\Delta p = 4 f \rho \frac{L}{D} \frac{v^2}{2gc} = 3500 \frac{lb_f}{ft^2}$$ (3.5-13) also (2.10-5)

Problem 5. Geankoplis, problem 3.6-1, page 209

constant density, flows in z direction through circular pipe with azial symmetry.
(a) use shell balance to derive continuity equation.

volume of our system:

$$A_z = \left[\pi (r + \Delta r)^2 - \pi (r)^2 \right] = 2 \pi r \Delta r$$

$$A_{r+\Delta r} = \left[2\pi (r + \Delta r) \right] \Delta z$$

$$A_r = \left[2\pi (r) \right] \Delta z$$

$$V = \left[\pi (r + \Delta r)^2 - \pi (r)^2 \right] \Delta z$$

$$V = \pi \Delta z \left[r^2 + 2 r \Delta r + \Delta r^2 - r^2 \right] = \pi \Delta z \left[2 r \Delta r + \Delta r^2 \right] = 2 \pi r \Delta z \Delta r$$

accumulation = in - out + gen - con

$$\text{gen} = \text{con} = 0$$

$$\text{acc} = V\frac{\partial \rho}{\partial t} = 2\pi r \Delta z \Delta r \frac{\partial \rho}{\partial t}$$

$$\text{in} = \text{in}_r + \text{in}_z = A_r \rho v_r \,|_r + A_z \rho v_z \,|_z$$
$$= [2\pi(r)]\ \Delta z \rho v_r \,|_r + 2\pi r \Delta r \rho v_z \,|_z$$

$$\text{out} = [2\pi(r + \Delta r)]\Delta z \rho v_r \,|_{r+\Delta r} + 2\pi r \Delta r \rho v_z \,|_{z+\Delta z}$$

Put these five terms in mass balance:

$$2\pi r \Delta z \Delta r \frac{\partial \rho}{\partial t} = [2\pi(r)]\ \Delta z \rho v_r \,|_r + 2\pi r \Delta r \rho v_z \,|_z$$
$$- [2\pi(r + \Delta r)]\Delta z \rho v_r \,|_{r+\Delta r} - 2\pi r \Delta r \rho v_z \,|_{z+\Delta z}$$

Divide by $2\pi \Delta z \Delta r$:

$$r\frac{\partial \rho}{\partial t} = \frac{r \rho v_r \,|_r}{\Delta r} + \frac{r \rho v_z \,|_z}{\Delta z}$$
$$- \frac{[(r + \Delta r)]\rho v_r \,|_{r+\Delta r}}{\Delta r} - \frac{r \rho v_z \,|_{z+\Delta z}}{\Delta z}$$

Rearrange into a form recognizable as the definition of a derivative:

$$-r\frac{\partial \rho}{\partial t} = \frac{[(r + \Delta r)]\rho v_r \,|_{r+\Delta r} - r \rho v_r \,|_r}{\Delta r} + \frac{- r \rho v_z \,|_{z+\Delta z}\ r \rho v_z \,|_z}{\Delta z}$$

Take limits as differential elements approach 0 and apply the definition of the derivative:

$$-r\frac{\partial \rho}{\partial t} = \frac{\partial (r \rho v_r)}{\partial r} + \frac{\partial (r \rho v_z)}{\partial z}$$

Consider that, density is constant and r is not a function of z, so

$$0 = \rho \frac{\partial (r v_r)}{\partial r} + r \rho \frac{\partial v_z}{\partial z}$$

$$0 = \frac{1}{r}\frac{\partial(rv_r)}{\partial r} + \frac{\partial v_z}{\partial z}$$

This is the equation of continuity for flow in a cylindrical pipe with axial symmetry for an incompressible fluid. You see that we have only 2 of the four terms in the most general form of the continuity equation in cylindrical coordinates as given in equation 3.6-27 on page 169, Geankoplis. We lost one term due to incompressibility and the other due to axial symmetry.

(b) Use the equation of continuity in cylindrical coordinates (3.6-27) to derive the equation.

$$-\frac{\partial \rho}{\partial t} = \nabla \cdot \rho \underline{v} = \frac{1}{r}\frac{\partial(\rho r v_r)}{\partial r} + \frac{1}{r}\frac{\partial(\rho v_\theta)}{\partial \theta} + \frac{\partial(\rho v_z)}{\partial z}$$

LHS = 0 because of incompressibility.
middle term of RHS = 0 because of axial symmetry.

$$0 = \frac{1}{r}\frac{\partial(\rho r v_r)}{\partial r} + \frac{\partial(\rho v_z)}{\partial z}$$

pull density out of derivative because of incompressibility.

$$0 = \frac{1}{r}\frac{\partial(r v_r)}{\partial r} + \frac{\partial(v_z)}{\partial z}$$

Voila!