Homework Assignment Number Four Solution
Assigned: Wednesday, February 3, 1999
Due: Wednesday, February 10 1999 BEFORE LECTURE STARTS.

Problem 1. Geankoplis, problem 2.8-4, page 110

We have water.

$$q = 0.050\frac{m^3}{s}$$

Expanding bend. $\alpha = 120^\circ$, $D_1 = 0.0762m$, $D_2 = 0.2112m$, $p_1 = 68{,}940Pa(gage)$
Neglect energy losses.

$$\bar{v}_1 = \frac{q}{A_1} = \frac{0.050}{\frac{\pi}{4}D_1^2} = 10.96\frac{m}{s}$$

$$\bar{v}_2 = \frac{q}{A_2} = \frac{0.050}{\frac{\pi}{4}D_2^2} = 1.43\frac{m}{s}$$

$$N_{Re1} = \frac{D_1\bar{v}_1\rho}{\mu} = \frac{0.0762\cdot 10.96\cdot 1000}{0.001} = 8351520$$

$$N_{Re2} = \frac{D_2\bar{v}_2\rho}{\mu} = \frac{0.2112\cdot 1.43\cdot 1000}{0.001} = 302016$$

turbulent.
Mechanical energy balance:

$$0 = \frac{\Delta p}{\rho} + \frac{\Delta v^2}{2\alpha} + g\Delta z + \hat{W}_s + \sum\hat{F}$$

$$0 = \frac{p_2 - 68940 - 101325}{1000} + \frac{1.43^2 - 10.96^2}{2} + 0 + 0 + 0$$

$$p_2 = 229303\,Pa(abs) = 127978\,Pa(gage)$$

From momentum balance on page 74:

$$R_x = \dot{m}\bar{v}_2\cos\alpha_2 - \dot{m}\bar{v}_1\cos\alpha_1 + p_2A_2\cos\alpha_2 - p_1A_1\cos\alpha_1$$

$$R_y = \dot{m}\bar{v}_2\sin\alpha_2 - \dot{m}\bar{v}_1\sin\alpha_1 + p_2A_2\sin\alpha_2 - p_1A_1\sin\alpha_1 + m_tg$$

$$\dot{m} = \rho\bar{v}_1A_1 = \rho\bar{v}_2A_2 = 50\frac{kg}{s}$$

We must realize that: $\alpha_1 = 0^\circ$ and $\alpha_2 = 120^\circ$. Then we know everything to plug into the above equations except m_t, the total mass inside the pipe, which we cannot calculate without knowing the volume inside the pipe.

$$R_x = 50 \cdot 1.43 \cdot (-0.5) - 50 \cdot 10.96 \cdot 1 +$$
$$229303 \cdot \frac{\pi}{4} D_2^2 \cdot (-0.5) - (68940 + 101325)\frac{\pi}{4} D_1^2 \cdot 1 = -5376.8\text{N}$$
$$R_y = 50 \cdot 1.43 \cdot (0.866) - 0 + 229303 \cdot \frac{\pi}{4} D_2^2 \cdot 0.866 - 0 + m_t(9.8)$$
$$R_y = 7018 + m_t(9.8)\text{N}$$

Problem 2. Geankoplis, problem 2.8-7, page 110

$$\bar{v}_1 = 30.5\frac{\text{m}}{\text{s}},\ D_1 = 0.01$$

Vane is U-shaped so $\alpha_1 = 0^\circ$ and $\alpha_2 = 180^\circ$

Ignore pressure terms for a free jet because pressure is the same everywhere.

$$R_x = \dot{m}\bar{v}_2 \cos\alpha_2 - \dot{m}\bar{v}_1 \cos\alpha_1$$

$$R_y = \dot{m}\bar{v}_2 \sin\alpha_2 - \dot{m}\bar{v}_1 \sin\alpha_1 + m_t g$$

We ignore the

$$\dot{m} = \rho\bar{v}_1 A_1 = 1000 \cdot 30.5 \cdot \frac{\pi}{4} 0.01^2 = 1000 \cdot 30.5 \cdot 7.85 \cdot 10^{-5} = 2.40\frac{\text{kg}}{\text{s}}$$

Assume that the cross-sectional area of flow is constant so
$\dot{m} = \rho\bar{v}_1 A_1 = \rho\bar{v}_2 A_2$ and $\bar{v}_1 = \bar{v}_2$, then

$$R_x = 2.40 \cdot 30.5 \cdot (-1) - 2.40 \cdot 30.5 \cdot (1) = -146.4\text{N}$$
$R_y = 2.40 \cdot 30.5 \cdot (0) - 2.40 \cdot 30.5 \cdot (0) + m_t g = 0$ if we neglect gravity.

Problem 3. Geankoplis, problem 2.9-2, page 110

constant density, laminar flow, steady state, horizontal between 2 flat parallel plates. Separated by a distance of $2y_o$.

$$v_x(y) = \frac{(p_0 - p_L)y_o^2}{2\mu L}\left[1 - \left(\frac{y}{y_o}\right)^2\right] \text{ for } -y_o < y < y_o$$

This derivation follows the derivation done in class for the fluid flowing down a plate.
The control volume is a rectangular cube of dimensions
$V = LW\Delta y$ where W is some arbitrary width in the z-direction, L is the length in the flow direction, x, and delta y is the incremental width in the direction perpendicular to the parallel plates.
Then the shell momentum balance becomes:

$$0 = LW\tau_{yx}\,|_{y+\Delta y} - LW\tau_{yx}\,|_{y} +$$
$$\Delta y W \bar{v}_y(\rho\bar{v})\,|_{z=L} - \Delta y W \bar{v}_y(\rho\bar{v})\,|_{z=0}$$
$$p\Delta y W\,|_{z=L} - p\Delta y W\,|_{z=0}$$

The two convective terms in the middle cancel. Divide by $LW\Delta y$

$$0 = \frac{\tau_{yx}\,|_{y+\Delta y} - \tau_{yx}\,|_{y}}{\Delta y} + \frac{p\,|_{z=L} - p\,|_{z=0}}{L}$$

$$\frac{d\tau_{yx}}{dy} = -\frac{\Delta p}{L}$$

Integrate, using the Boundary Condition given in the problem statement.

$$\int_{\tau_{yx(y=0)}}^{\tau_{yx(y)}} d\tau_{yx} = -\int_{y=0}^{y} \frac{\Delta p}{L} dy = -\frac{\Delta p}{L}\int_{y=0}^{y} dy$$

$$\tau_{yx} = -\frac{\Delta p}{L} y$$ (LINEAR PROFILE OF STRESS)

Remember Newton's Law of Viscosity:

$$\tau_{yx} = -\mu\frac{dv_x}{dy}$$

$$-\mu\frac{dv_x}{dy} = -\frac{\Delta p}{L} y$$

$$\mu\int_{v_{x(y=0)}}^{v_{x(y)}} dv_x = \frac{\Delta p}{L}\int_{y=0}^{y} y\,dy$$

$$\mu\int_{v_{x(y)}}^{v_{x(y_o)}} dv_x = \frac{\Delta p}{L}\int_{y}^{y_o} y\,dy$$

$$\mu\left(v_x(y_o) - v_x(y)\right) = \frac{\Delta p}{L}\left(\frac{{y_o}^2}{2} - \frac{y^2}{2}\right)$$

$$\left(0 - v_x(y)\right) = \frac{\Delta p}{2\mu L}{y_o}^2\left(1 - \frac{y^2}{{y_o}^2}\right)$$

$$v_x(y) = -\frac{\Delta p}{2\mu L} y_o^2 \left(1 - \frac{y^2}{y_o^2}\right) = -\frac{(p_0 - p_L)}{2\mu L} y_o^2 \left(1 - \frac{y^2}{y_o^2}\right)$$ PARABOLIC PROFILE

Problem 4. Geankoplis, problem 2.10-1, page 111

Use Hagen-Poiseuille eqn to measure viscosity;

$$\bar{v} = \frac{\Delta p D^2}{32\mu L}$$ Hagen-Poiseuille Equation,

$$\mu = \frac{\Delta p D^2}{32\bar{v}L}$$

$$\Delta p = \rho g h = 996 \cdot 9.8 \cdot 0.131 = 1289.1 Pa$$

$$\bar{v} = \frac{q}{A} = \frac{q}{\frac{\pi}{4}D^2} = \frac{5.33 \cdot 10^{-7}}{\frac{\pi}{4}0.002222^2} = 0.13745 \frac{m}{s}$$

$$\mu = \frac{1289.1 \cdot 0.002222^2}{32 \cdot 0.13745 \cdot 0.1585} = 0.009129\ kg/m/s$$