Homework Assignment Number Three Solutions
Assigned: Wednesday, January 27, 1999
Due: Wednesday, February 3 1999 BEGINNING OF CLASS.

Problem 1. Geankoplis, problem 2.6-4, page 106

$$v_x(y) = v_{max}\left[1-\left(\frac{y}{y_o}\right)^2\right] \text{ for } -y_o < y < y_o$$

$$\bar{v} = \frac{\iint_A v\,dA}{\iint_A dA} = \frac{\int_{-y_o}^{y_o}\int_0^W v_x(y)\,dz\,dy}{\int_{-y_o}^{y_o}\int_0^W dz\,dy} = \frac{W\int_{-y_o}^{y_o} v_x(y)\,dy}{2y_oW}$$

$$\bar{v} = \frac{\int_{-y_o}^{y_o} v_{max}\left[1-\left(\frac{y}{y_o}\right)^2\right]dy}{2y_o} = \frac{v_{max}\left[y-\left(\frac{y^3}{3y_o{}^2}\right)\right]_{-y_o}^{y_o}}{2y_o}$$

$$\bar{v} == \frac{2v_{max}\left[y_o-\left(\frac{y_o{}^3}{3y_o{}^2}\right)\right]}{2y_o} = \frac{2v_{max}}{3}$$

Problem 2. Geankoplis, problem 2.6-6, page 107
If we consider a tank as shown in Figure 2..6-5 on page 54, we can write the total mass balance:

$$acc = in - out +/- gen/con$$

Draw the system. Define the control volume.
If we assume no reaction:
Constant flow rates in and out.

$$acc = in - out$$

$$\frac{dm(t)}{dt} = \dot{m}_{in} - \dot{m}_{out}$$

$$\int_{m(t'=0)}^{m(t'=t)} dm(t') = \int_{t'=0}^{t'=t}\left(\dot{m}_{in} - \dot{m}_{out}\right)dt$$

$$m(t) - m(t=0) = (\dot{m}_{in} - \dot{m}_{out})t$$

$$m(t) = (\dot{m}_{in} - \dot{m}_{out})t + m(t=0)$$

For the problem specifications:

$$m(t) = (900 - 600)t + 500 = 300t + 500 \text{ kg}$$

Now look at the weight fraction of salt in the tank by writing a salt balance.

$$acc = in - out$$

$$\frac{dm(t)w_s(t)}{dt} = \dot{m}_{in}w_{s,in} - \dot{m}_{out}w_{s,out}$$

$$w_s(t)\frac{dm(t)}{dt} + m(t)\frac{dw_s(t)}{dt} = \dot{m}_{in}w_{s,in} - \dot{m}_{out}w_{s,out}$$

Substitute our result from above for the mass in the tank and the change in mass in the tank.

$$\frac{dm(t)}{dt} = \dot{m}_{in} - \dot{m}_{out}$$

$$m(t) = (\dot{m}_{in} - \dot{m}_{out})t + m(t=0)$$

$$w_s(t)(\dot{m}_{in} - \dot{m}_{out}) + [(\dot{m}_{in} - \dot{m}_{out})t + m(t=0)]\frac{dw_s(t)}{dt} = \dot{m}_{in}w_{s,in} - \dot{m}_{out}w_{s,out}$$

$$[(\dot{m}_{in} - \dot{m}_{out})t + m(t=0)]\ dw_s(t) = [(\dot{m}_{in}w_{s,in} - \dot{m}_{out}w_{s,out}) - w_s(t)(\dot{m}_{in} - \dot{m}_{out})]dt$$

$$\frac{dw_s(t)}{[(\dot{m}_{in}w_{s,in} - \dot{m}_{out}w_{s,out}) - w_s(t)(\dot{m}_{in} - \dot{m}_{out})]} = \frac{dt}{[(\dot{m}_{in} - \dot{m}_{out})t + m(t=0)]}$$

$$\int_{w_s(t'=0)}^{w_s(t'=t)} \frac{dw_s(t')}{[(\dot{m}_{in}w_{s,in} - \dot{m}_{out}w_{s,out}) - w_s(t')(\dot{m}_{in} - \dot{m}_{out})]} = \int_{t'=0}^{t'=t} \frac{dt}{[(\dot{m}_{in} - \dot{m}_{out})t + m(t=0)]}$$

Assume tank is well mixed so:

$$w_{s,out} = w_s(t')$$

$$\int_{w_s(t'=0)}^{w_s(t'=t)} \frac{dw_s(t')}{\left[\dot{m}_{in} w_{s,in} - w_s(t')\dot{m}_{in}\right]} = \int_{t'=0}^{t'=t} \frac{dt}{\left[(\dot{m}_{in} - \dot{m}_{out})t + m(t=0)\right]}$$

$$\frac{-1}{\dot{m}_{in}} \ln\left[\frac{w_{s,in} - w_s(t)}{w_{s,in} - w_s(t=0)}\right] = \frac{1}{(\dot{m}_{in} - \dot{m}_{out})} \ln\left[\frac{(\dot{m}_{in} - \dot{m}_{out})t + m(t=0)}{m(t=0)}\right]$$

$$\ln\left[\frac{w_{s,in} - w_s(t)}{w_{s,in} - w_s(t=0)}\right] = \ln\left[\left(\frac{(\dot{m}_{in} - \dot{m}_{out})t + m(t=0)}{m(t=0)}\right)^{\frac{-\dot{m}_{in}}{(\dot{m}_{in} - \dot{m}_{out})}}\right]$$

$$\frac{w_{s,in} - w_s(t)}{w_{s,in} - w_s(t=0)} = \left(\frac{(\dot{m}_{in} - \dot{m}_{out})t + m(t=0)}{m(t=0)}\right)^{\frac{-\dot{m}_{in}}{(\dot{m}_{in} - \dot{m}_{out})}}$$

$$w_s(t) = w_{s,in} - \left(w_{s,in} - w_s(t=0)\right)\left(\frac{m(t=0)}{(\dot{m}_{in} - \dot{m}_{out})t + m(t=0)}\right)^{\frac{\dot{m}_{in}}{(\dot{m}_{in} - \dot{m}_{out})}}$$

From the problem specifications:

$$w_s(t) = 0.1667 - (0.1667 - 0.05)\left(\frac{500}{300t + 500}\right)^{\frac{900}{300}}$$

$$w_s(t) = 0.1667 - 0.1167\left(\frac{5}{3t + 5}\right)^3$$

$$w_s(t=2) = 0.1667 - 0.1167\left(\frac{5}{3(2) + 5}\right)^3 = 0.1557 \text{ weight percent salt}$$

Problem 3. Geankoplis, problem 2.7-2, page 107

$$v_x(y) = v_{max}\left[1 - \left(\frac{y}{y_o}\right)^2\right] \text{ for } -y_o < y < y_o$$

$$\bar{v} = \frac{2v_{max}}{3}, \ \bar{v}^3 = \frac{8{v_{max}}^3}{27}$$

$$\overline{v^3} = \frac{\iint_A v^3 dA}{\iint_A dA} = \frac{\int_{-y_o}^{y_o}\int_0^W {v_x}^3 dzdy}{\int_{-y_o}^{y_o}\int_0^W dzdy} = \frac{W\int_{-y_o}^{y_o} {v_x}^3 dy}{2y_o W} = \frac{\int_{-y_o}^{y_o} {v_x}^3 dy}{2y_o}$$

$${v_x}^3 = {v_{max}}^3\left[1 - 3\left(\frac{y}{y_o}\right)^2 + 3\left(\frac{y}{y_o}\right)^4 - \left(\frac{y}{y_o}\right)^6\right]$$

$$\overline{v^3} = \frac{{v_{max}}^3}{2y_o}\left[y - \frac{3}{3}\left(\frac{y^3}{{y_o}^2}\right) + \frac{3}{5}\left(\frac{y^5}{{y_o}^4}\right) - \frac{1}{7}\left(\frac{y^7}{{y_o}^6}\right)\right]_{-y_o}^{y_o}$$

$$\overline{v^3} = \frac{{v_{max}}^3}{y_o}\left[y_o - \frac{3}{3}\left(\frac{{y_o}^3}{{y_o}^2}\right) + \frac{3}{5}\left(\frac{{y_o}^5}{{y_o}^4}\right) - \frac{1}{7}\left(\frac{{y_o}^7}{{y_o}^6}\right)\right] = \frac{16}{35}{v_{max}}^3$$

$$\alpha = \frac{\bar{v}^3}{\overline{v^3}} = \frac{\frac{8{v_{max}}^3}{27}}{\frac{16}{35}{v_{max}}^3} = \frac{35}{54}$$

Problem 4. Geankoplis, problem 2.7-4, page 107

$$\frac{\partial\left[\rho V\left(U + \frac{v^2}{2} + zg\right)\right]}{\partial t} = \left[\rho v A\left(H + \frac{v^2}{2\alpha} + zg\right)\right]_{in} - \left[\rho v A\left(H + \frac{v^2}{2\alpha} + zg\right)\right]_{out} + Q - \dot{W}_s + \sum F$$

Neglect Kinetic energy. Assume Steady state. Neglect friction.

$$\dot{m} = \rho\bar{v}_1 A_1 = \rho\bar{v}_2 A_2 = \rho q = 1000\frac{kg}{m^3}\cdot 0.189\frac{m^3}{min} = 189\frac{kg}{min} = 3.15\frac{kg}{s}$$

$$z_2 - z_1 = 15.24$$

$$Q = -704kW = -704{,}000 J/s$$

$$\dot{W}_s = -1.49\text{kW} = -1490\text{J/s}$$

$H_1 = 390{,}582\text{J/kg}$ from steam table, linear interpolation

Overall energy balance becomes:

$$0 = -3.15\left[\left(H_2 - 390582 + 9.8(15.24)\right)\right] - 704000 + 1490$$

$$H_2 = 167414$$

From the steam tables, linear interpolation, the outlet temperature is: $T_2 = 40.0$

The enthalpy gain due to the work is: 1490 J/s

$$\frac{1490\text{J/s}}{3.15\frac{\text{kg}}{\text{s}}} = 473\frac{\text{J}}{\text{kg}}$$

Problem 5. Geankoplis, problem 2.7-8, page 108

$$\dot{m} = \rho\bar{v}_1 A_1 = \rho\bar{v}_2 A_2 = \rho q = 1150\frac{\text{kg}}{\text{m}^3}\cdot 0.2\frac{\text{ft}^3}{\text{s}}\left(\frac{\text{m}}{3.2808\text{ft}}\right)^3 = 6.513\frac{\text{kg}}{\text{s}}$$

$$\bar{v}_1 = \frac{\dot{m}}{\rho A_1} = \frac{6.513}{1150\cdot\frac{\pi}{4}\cdot\left(3.548\text{in}\cdot\frac{0.0254\text{m}}{\text{in}}\right)^2} = 0.8879\frac{\text{m}}{\text{s}}$$

$$\bar{v}_2 = \frac{\dot{m}}{\rho A_2} = \frac{6.513}{1150\cdot\frac{\pi}{4}\cdot\left(2.067\text{in}\cdot\frac{0.0254\text{m}}{\text{in}}\right)^2} = 2.616\frac{\text{m}}{\text{s}}$$

Energy balance just around pump--nothing but pressure, kinetic energy, and work terms:

$$0 = \frac{\Delta p}{\rho} + \frac{\Delta v^2}{2\alpha} + g\Delta z + \hat{\dot{W}}_s + \sum\hat{F}$$

$$\Delta p = -\rho\left(\frac{\Delta v^2}{2\alpha} + \hat{\dot{W}}_s\right)$$

Need the work. Solve mechanical energy balance around entire system:

$$0 = \frac{\Delta p}{\rho} + \frac{\Delta v^2}{2\alpha} + g\Delta z + \hat{\dot{W}}_s + \sum\hat{F}$$

$$0 = 0 + \frac{2.616^2 - 0.8879^2}{2} + 9.8(75\text{ft})\cdot\left(\frac{\text{m}}{3.2808\text{ft}}\right) + \hat{\dot{W}}_s + 18\frac{\text{ft}\cdot\text{lb}_f}{\text{lb}_f}\cdot\left(2.9890\frac{\text{J/kg}}{\frac{\text{ft}\cdot\text{lb}_f}{\text{lb}_f}}\right)$$

$\hat{\dot{W}}_s = -3.0275 - 224.03 - 53.802 = -280.86$ Joules/kg done on system.

$$\hat{\dot{W}}_s = \eta \hat{\dot{W}}_p \text{ so } \hat{\dot{W}}_p = \frac{\hat{\dot{W}}_s}{\eta} = \frac{-280.86}{0.7} = -401.3\frac{J}{kg} \cdot 6.513\frac{kg}{s} = -2613 \text{Watts}$$

$$\hat{\dot{W}}_p = 2613 \text{Watts} \cdot \frac{hp}{745.70W} = 3.50hp$$

Now, we can go back and determine what was the pressure developed across the pump:

$$\Delta p = -\rho\left(\frac{\Delta v^2}{2\alpha} + \hat{\dot{W}}_s\right) = -1150\left(\frac{2.616^2 - 0.8879^2}{2} + -401.3\right) = 458013Pa$$

$$\Delta p = 458013Pa \cdot \frac{1atm}{101325Pa} = 4.52atm$$

Problem 6. Geankoplis, problem 2.7-9, page 108

(a) horizontal flow

$$\dot{m} = \rho \bar{v}_1 A_1 = 998\frac{kg}{m^3} \cdot 1.676\frac{m}{s}\frac{\pi}{4} \cdot \left(3.068in \cdot \frac{0.0254m}{in}\right)^2 = 7.978\frac{kg}{s}$$

$$\bar{v}_2 = \frac{\dot{m}}{\rho A_2} = \frac{7.978}{998 \cdot \frac{\pi}{4} \cdot \left(2.067in \cdot \frac{0.0254m}{in}\right)^2} = 3.693\frac{m}{s}$$

Energy balance, neglect friction, pipe is horizontal:

$$0 = \frac{\Delta p}{\rho} + \frac{\Delta v^2}{2\alpha} + g\Delta z + \hat{\dot{W}}_s + \sum \hat{F}$$

$$\frac{\Delta p}{\rho} = -\frac{\Delta v^2}{2\alpha}$$

$$p_2 = p_1 - \rho\frac{\Delta v^2}{2\alpha} = 68900Pa - 998\frac{\left(3.693^2 - 1.676^2\right)}{2} = 63496 \text{ Pa}$$

(b) vertical flow

$$0 = \frac{\Delta p}{\rho} + \frac{\Delta v^2}{2\alpha} + g\Delta z + \hat{\dot{W}}_s + \sum \hat{F}$$

$$\frac{\Delta p}{\rho} = -\frac{\Delta v^2}{2\alpha} - g\Delta z$$

$$p_2 = p_1 - \rho\left(\frac{\Delta v^2}{2\alpha} + g\Delta z\right) = 68900 - 998\left[\frac{\left(3.693^2 - 1.676^2\right)}{2} + 9.8 \cdot 0.457\right] = 59027 \text{ Pa}$$

Problem 7. Geankoplis, problem 2.7-11, page 109

$$\dot{m} = \rho \bar{v}_1 A_1 = \rho \bar{v}_2 A_2 = \rho q = 1000 \frac{kg}{m^3} \cdot 0.800 \frac{m^3}{s} = 800 \frac{kg}{s}$$

$$z_2 - z_1 = -5 - 89.5 = -94.5 \text{ m}$$

$$p_2 - p_1 = 89600 - 172400 = -82800 \text{ Pa}$$

$$\bar{v}_2^{\,2} - \bar{v}_1^{\,2} = 0.0$$

$$\dot{W}_s = \frac{658000 J/s}{0.89} = 739326 J/s$$

$$0 = \frac{\Delta p}{\rho} + \frac{\Delta v^2}{2\alpha} + g\Delta z + \hat{\dot{W}}_s + \sum \hat{F}$$

$$0 = \frac{-82800}{1000} + 0 + 9.8 \cdot (-94.5) + \frac{739326 J/s}{800 kg/s} + \sum \hat{F}$$

$$\sum \hat{F} = 84.7 \text{ J/kg}$$