

Homework Assignment Number Fourteen**Assigned: Wednesday, April 28, 1999****Due: Monday, May 3, 1999 BEFORE LECTURE STARTS.****Problem 1.**

Consider the slab of finite thickness of $2H$ in the x -direction. Consider heat conduction without generation through this slab. The temperature is held constant on both sides of the plate. The initial temperature is constant through-out the plate. The properly posed problem is thus:

$$\frac{\partial T}{\partial t} = \alpha \left(\frac{\partial^2 T}{\partial x^2} \right), \quad T(x, t = 0) = T_o, \quad T(x = 0, t) = T_1, \quad T(x = 2H, t) = T_1$$

For this problem, use $T_o = 300K$, $T_1 = 400K$, $H = 0.25m$, $\alpha = 9.8e-5 \frac{m^2}{s}$

(a) Find the midpoint temperature at 200 seconds using the analytical solution

$$Y(x, t) = \frac{T_1 - T(x, t)}{T_1 - T_o} = \frac{4}{\pi} \sum_{n=1,3,5}^{\infty} \left[\frac{1}{n} \exp\left(\frac{-n^2 \pi^2 \alpha t}{4H^2}\right) \sin\left(\frac{n\pi x}{2H}\right) \right]$$

where x ranges from 0 to $2H$ in this equation. Specify what value of n , you used to approximate infinity.

(b) Find the midpoint temperature at 200 seconds using MATLAB.

(c) Compare (a) and (b).

Problem 2.

Consider the slab of finite thickness of $2H$ in the x -direction. Consider heat conduction without generation through this slab. The temperature is held constant on one side of the plate. The other side of the plate is insulated. The initial temperature is constant through-out the plate. There is an infinitely thin surface film over the side of the plate that is not insulated. The properly posed problem is thus:

$$\frac{\partial T}{\partial t} = \alpha \left(\frac{\partial^2 T}{\partial x^2} \right), \quad T(x, t = 0) = T_o, \quad \frac{dT}{dx}(x = 0, t) = 0, \quad T(x = 2H, t) = T_1$$

For this problem, use $T_o = 300K$, $T_1 = 400K$, $H = 0.0231m$, $k = 0.197 \frac{W}{m \cdot K}$,

$$h = 8.52 \frac{W}{m^2 \cdot K}, \quad \rho = 998 \frac{kg}{m^3}, \quad \hat{C}_p = 2300 \frac{J}{kg \cdot K}$$

(a) Find the midpoint temperature at 200 HOURS using the chart on page 341 of Geankoplis.

Note that in this chart, $x_1 = 2H$, the full width of the slab, which contradicts the picture on page 338, but which is correct. The picture is bad.